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Shanghai Astronomical Observatory,Chinese Academy of Sciences

Detecting Cosmic 21 cm global signal using an improved polynomial fitting algorithm

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Shanghai Astronomical Observatory

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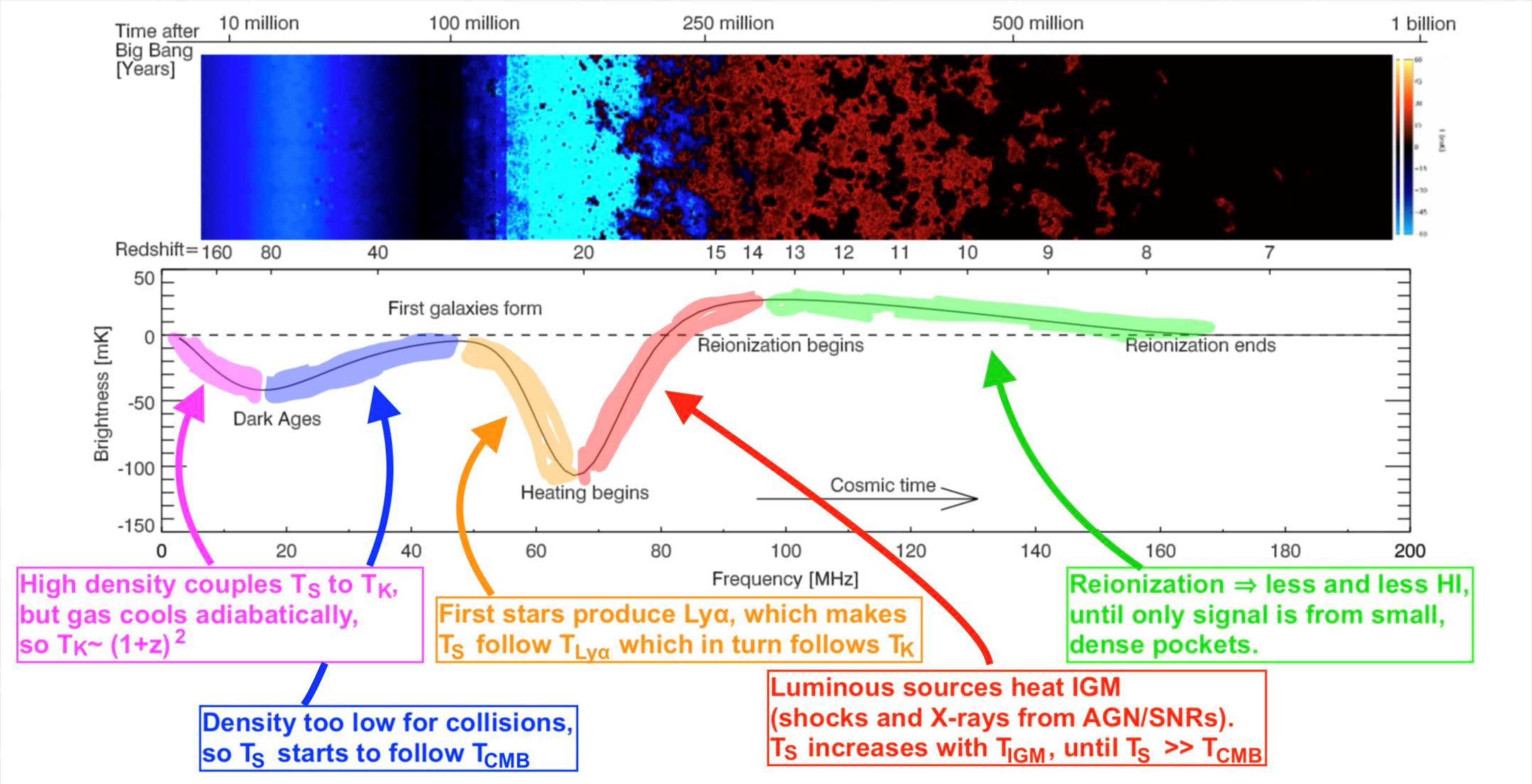
1. Background



J R Pritchard and A Loeb (2012)

1. Background

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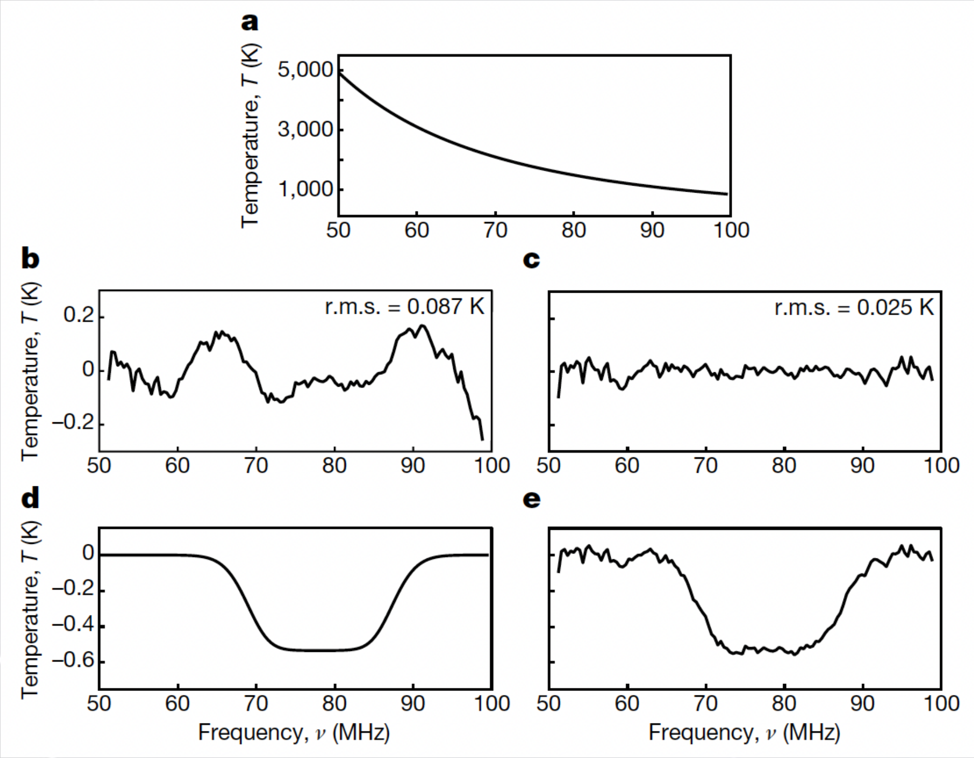
Global spectrum

EDGES



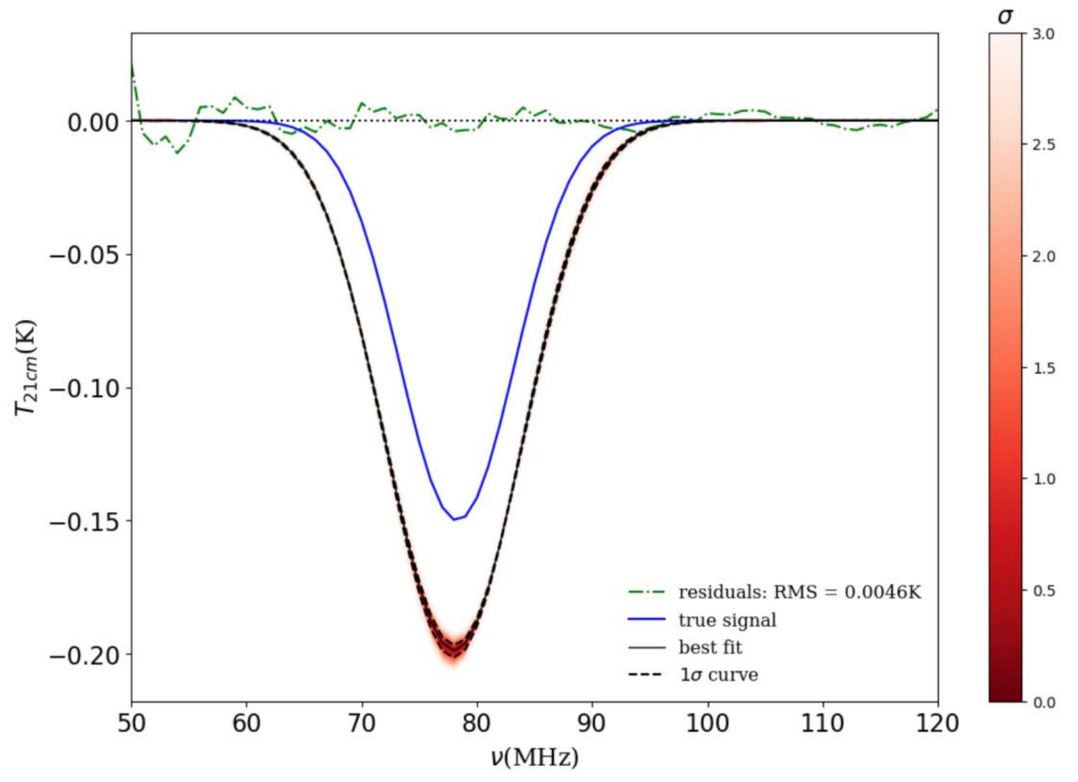
J D Bowman et al. (2012)

1. Background

Polynomial fit

Polynomial fit (5-order)

$$\hat{T}_A(\nu) = \hat{T}_A(\nu_r)\exp\left[\sum_{n=1}^{N} a_n \log^n\left(\frac{\nu}{\nu_r}\right)\right],$$



Questions:

- Chromaticity
- Spatial correlation of foreground spectral indices

Can we inverse the structure caused by the chromaticity in case we bring the beam information into the polynomial model?

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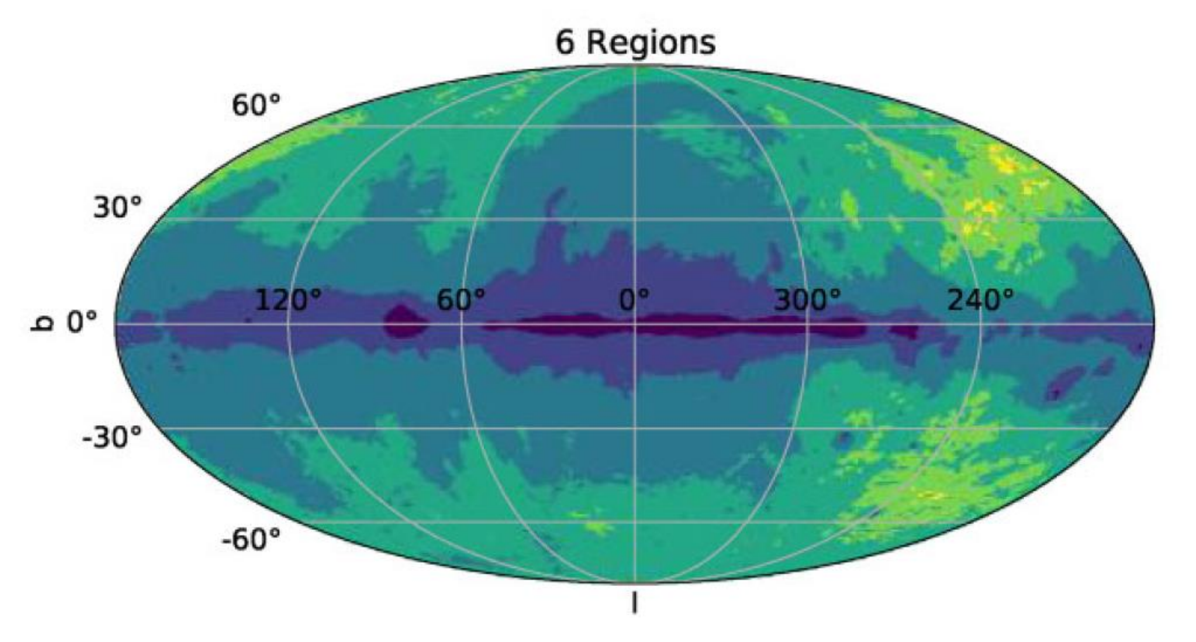
Method of chromaticity correction

Beam correction factor

$$C = \frac{\oiint T_b(\nu_r, \mathbf{n}) B_A(\nu, \mathbf{n}) \mathrm{d}\Omega}{\oiint T_b(\nu_r, \mathbf{n}) B_A(\nu_r, \mathbf{n}) \mathrm{d}\Omega},$$

Method raised by REACH

$$T_{\text{model}}(\nu) = \frac{1}{4\pi} \int_0^{4\pi} D(\theta, \phi, \nu) \times \int_{t_{\text{start}}}^{t_{\text{end}}} \sum_{i=1}^{N} M_i(\theta, \phi)(T_{230}(\theta, \phi) - T_{\text{CMB}}) \left(\frac{\nu}{230}\right)^{-\beta_i} \mathrm{d}t \mathrm{d}\Omega + T_{\text{CMB}}.$$



D Anstey et al. (2021)

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An improved polynomial fitting algorithm – VZOP

Common polynomial

$$T_A(\nu) = \exp\left[a_0 + \sum_{n=1}^{N} a_n \log^n\left(\frac{\nu}{\nu_r}\right)\right],$$

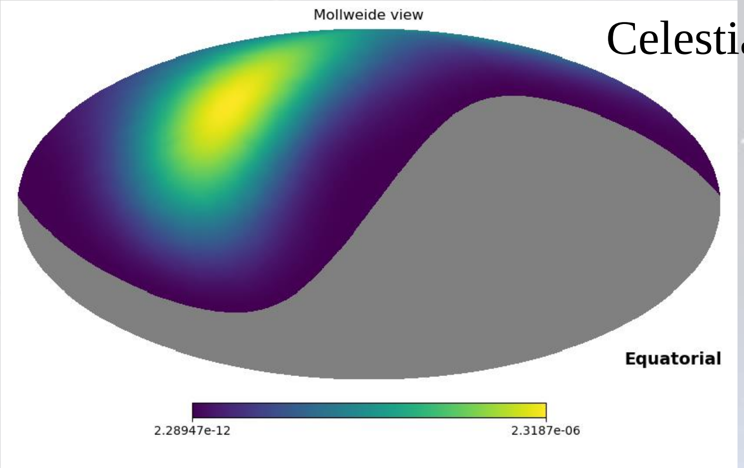
Vari-Zeroth-Order Polynomial (VZOP)

$$T_A(\nu) = \exp\left[a_0(\nu) + \sum_{n=1}^{N} a_n \log^n\left(\frac{\nu}{\nu_r}\right)\right].$$

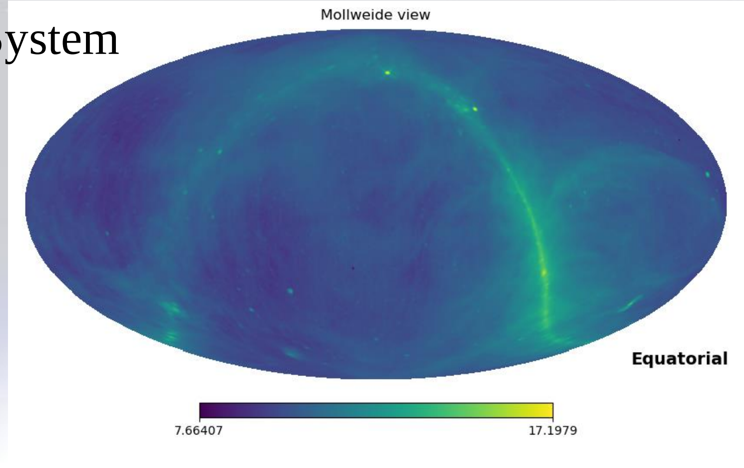
Degree of freedom of a_0 equals to the number of “declination bins,” which will be defined on the next two pages.

2. VZOP

Celestial Coordinate System



Convolutes
*

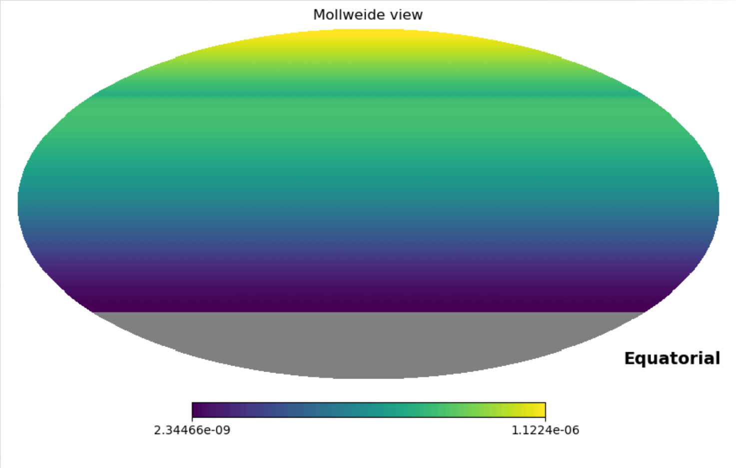


= Antenna temperature

Averaging them over the right ascension

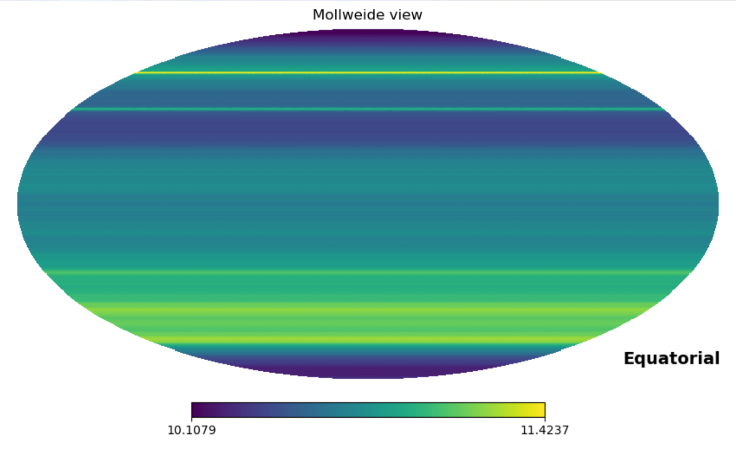
||

24-h averaged beam model



Times
X

24-h averaged temperature model



= Antenna temperature

An improved polynomial fitting algorithm - VZOP

θ is declination, ϕ is right ascension。

- $\bar{T}_b(\nu,\theta)$, 24-h averaged beam model
- $\bar{B}(\nu,\theta)$, 24-h averaged temperature model

In the Celestial Coordinate System

$$\log p(\mathbf{a}, \hat{\mathbf{T}}_{\rm gal}, \mathbf{p}_{\rm eor}|\mathbf{T}_A) = -\frac{N_\nu \log(2\pi) - \log(\det\mathbf{\Sigma})}{2} - \frac{\left(\mathbf{T}_A - \mathbf{S}(\mathbf{a})\mathbf{B}\hat{\mathbf{T}}_{\rm gal} - \hat{\mathbf{T}}_{\rm eor}\right)^T \mathbf{\Sigma}^{-1}\left(\mathbf{T}_A - \mathbf{S}(\mathbf{a})\mathbf{B}\hat{\mathbf{T}}_{\rm gal} - \hat{\mathbf{T}}_{\rm eor}\right)}{2},$$

B is a full-rank matrix with no more columns than rows.

$$\bar{T}_A(\nu) = \frac{1}{2\pi}\iiint B(\nu,\theta,\phi)T_b(\nu,\theta,\phi-\phi')\cos\theta\,\mathrm{d}\theta\mathrm{d}\phi\mathrm{d}\phi' = \int \bar{T}_b(\nu,\theta)\bar{B}(\nu,\theta)\cos\theta\,\mathrm{d}\theta,$$

Discretizing

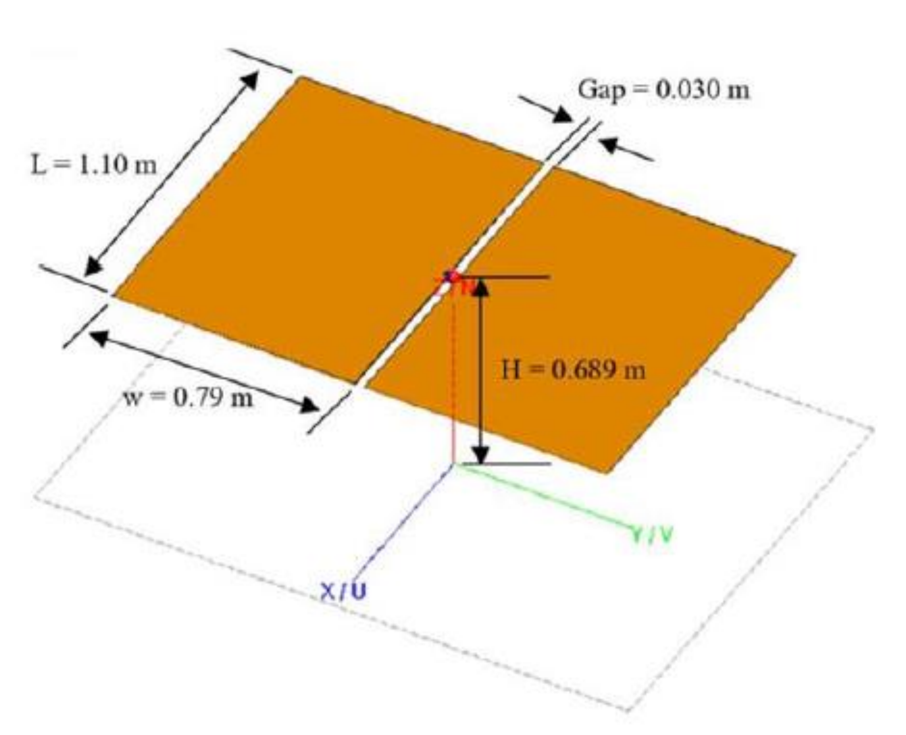
Divide declination into N bins

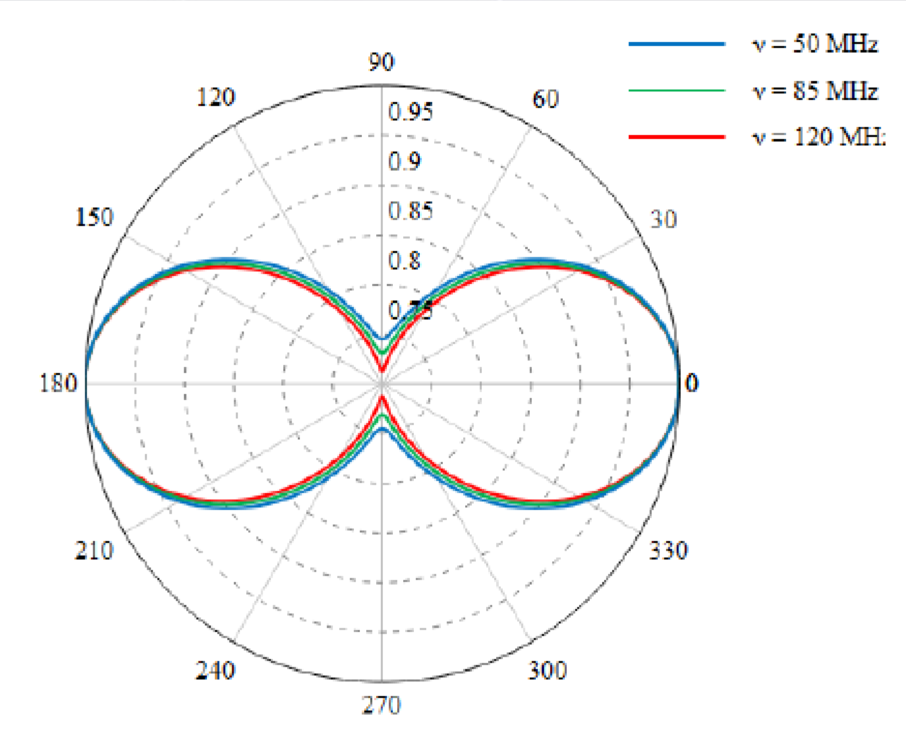
$$\hat{\bar{T}}_A(\nu_i) = \int \hat{\bar{T}}_b(\nu_i,\theta)\bar{B}(\nu_i,\theta)\cos\theta\,\mathrm{d}\theta \approx \sum_j \hat{\bar{T}}_{\rm gal}(\nu_r,\theta_j)S(\nu_i;\nu_r,\mathbf{a})\bar{B}(\nu_i,\theta_j)\cos\theta_j + \hat{\bar{T}}_{\rm eor}(\nu_i).$$

$$S(\nu;\nu_r,\mathbf{a}) \equiv \exp\left[\sum_{n=1}^{N} a_n \log^n\left(\frac{\nu}{\nu_r}\right)\right].$$

$$\hat{\bar{T}}_A(\nu_i) = \exp\left[a_0(\nu_i) + \sum_{n=1}^{N} a_n \log^n\left(\frac{\nu_i}{\nu_r}\right)\right] + \hat{\bar{T}}_{\rm eor}(\nu_i).$$

2. VZOP





- In our simulation, the antenna is placed by the 21CMA station (42.93°N, 86.68°E).

3. Simulation

- **Frequency range**: 50 – 120 MHz
- **Foreground model**: Galactic Global Sky Model (GSM, de Oliveira-Costa et al. 2008)
- **21cm signal model**:
  1. Gaussian model
  2. EDGES model
- **Thermal noise**:

$$T_{\rm eor}(\nu) = A \exp\left[-\frac{(\nu-\nu_c)^2}{2\omega^2}\right],$$

- amplitude: $A = -0.150$ K
- center frequency: $\nu_c = 78.3$ MHz
- width: $\omega = 5$ MHz.

$$T_{\rm eor}(\nu) = A\left(\frac{1-{\rm e}^{-\tau {\rm e}^{B}}}{1-{\rm e}^{-\tau}}\right),$$

where

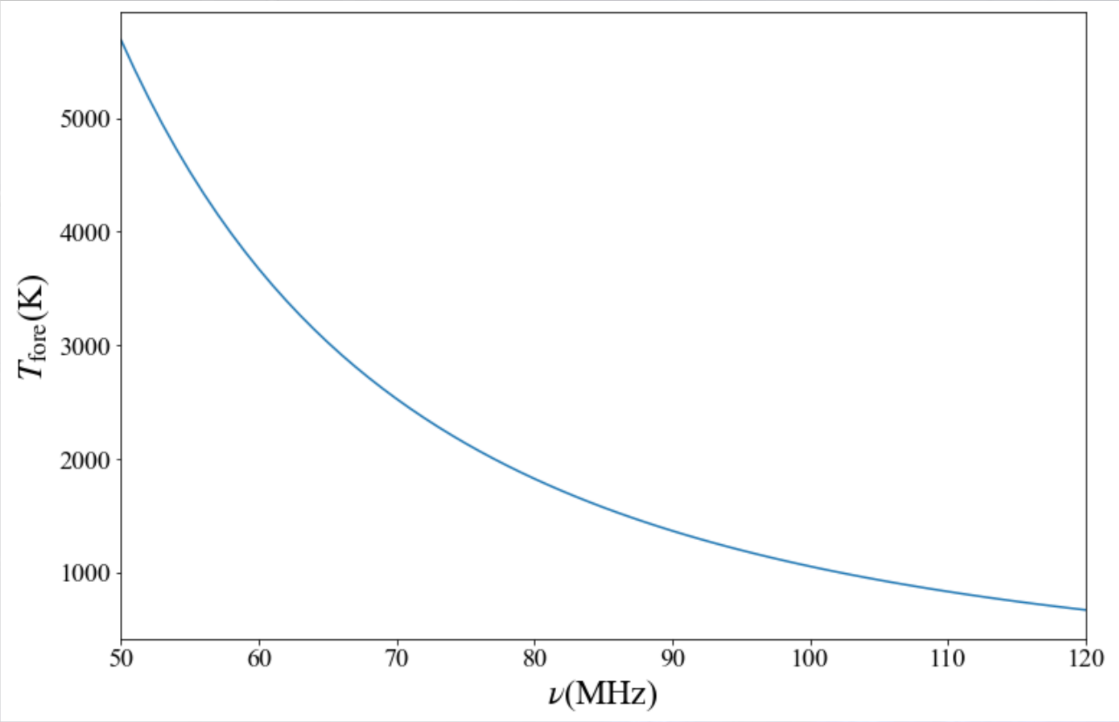
$$B = \frac{4(\nu-\nu_c)^2}{\omega^2}\log\left[-\frac{1}{\tau}\log\left(\frac{1+{\rm e}^{-\tau}}{2}\right)\right]$$

- amplitude: $A = -0.520$ K
- center frequency: $\nu_c = 78.3$ MHz
- fullwidth at half-maximum: $\omega = 20.3$ MHz
- flattening factor: $\tau = 7.$

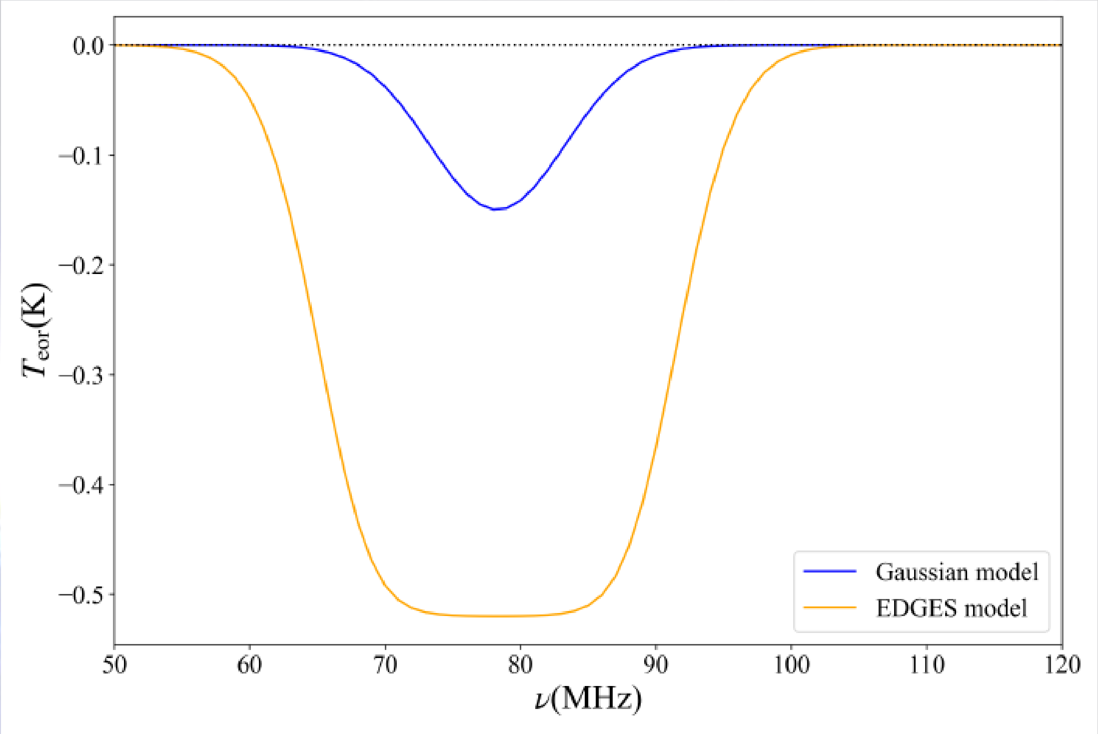
$$\sigma_n = \frac{T_A}{\sqrt{N \Delta\nu t_{\rm int}}}$$

- Number of antennas: $N = 1$
- channel bandwidth: $\Delta\nu = 1$ MHz
- integration time: $t_{\rm int} = 10$ d

3. Simulation



Antenna temperature based on the GSM



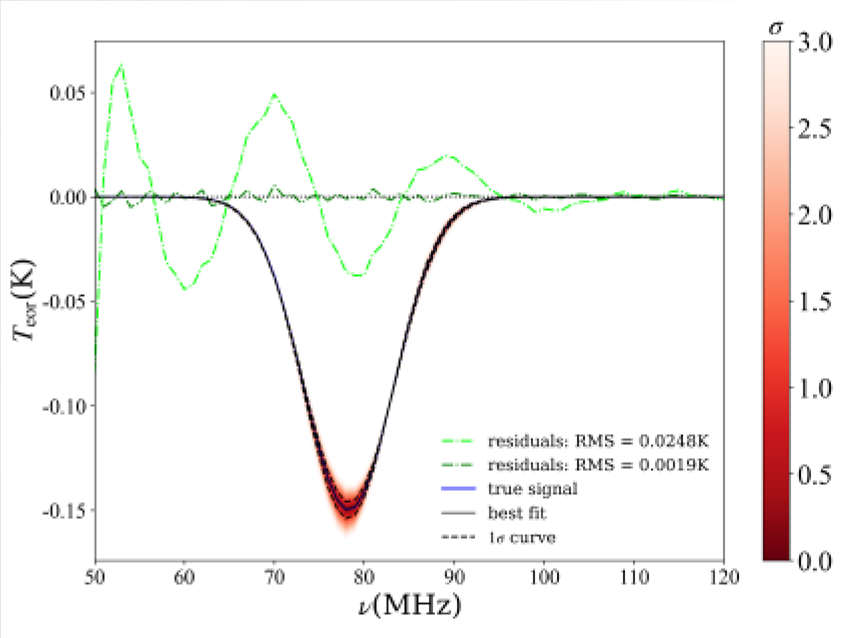
21cm signal model

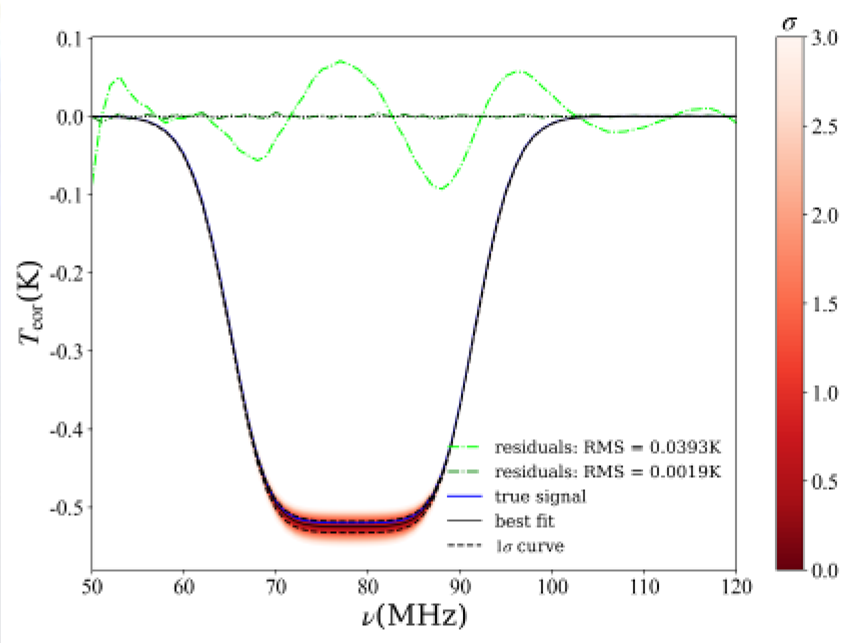
4. Results

- 5-order polynomial
- 10 declination bins

Fitting results

Assuming the antenna beam can be accurately measured





Inaccurate beam measurement

Completely random at different spatial positions but satisfying the following relations at different frequencies.

Model	Function
constant	$e_B(\nu, \mathbf{n}_0) = e_B(\nu_0, \mathbf{n}_0)$
linearity	$e_B(\nu, \mathbf{n}_0) = \left(\frac{\nu - 85}{35}\right) \times e_B(\nu_0, \mathbf{n}_0)$
quadratic	$e_B(\nu, \mathbf{n}_0) = \left[2\left(\frac{\nu - 85}{35}\right)^2 - 1\right] \times e_B(\nu_0, \mathbf{n}_0)$
cosine	$e_B(\nu, \mathbf{n}_0) = \cos\left(\frac{2\pi}{10}\nu\right) \times e_B(\nu_0, \mathbf{n}_0)$

Functions of four error models. Where e_B is the relative error between measured and real value, $\mathbf{n}_0$ is any fixed position and ν_0 = 50 MHz. Unit of frequency ν in each function is MHz.

Results: For all error models, using 10 bins can effectively extract 21 cm signal even if the errors reach 10%.

4. Results

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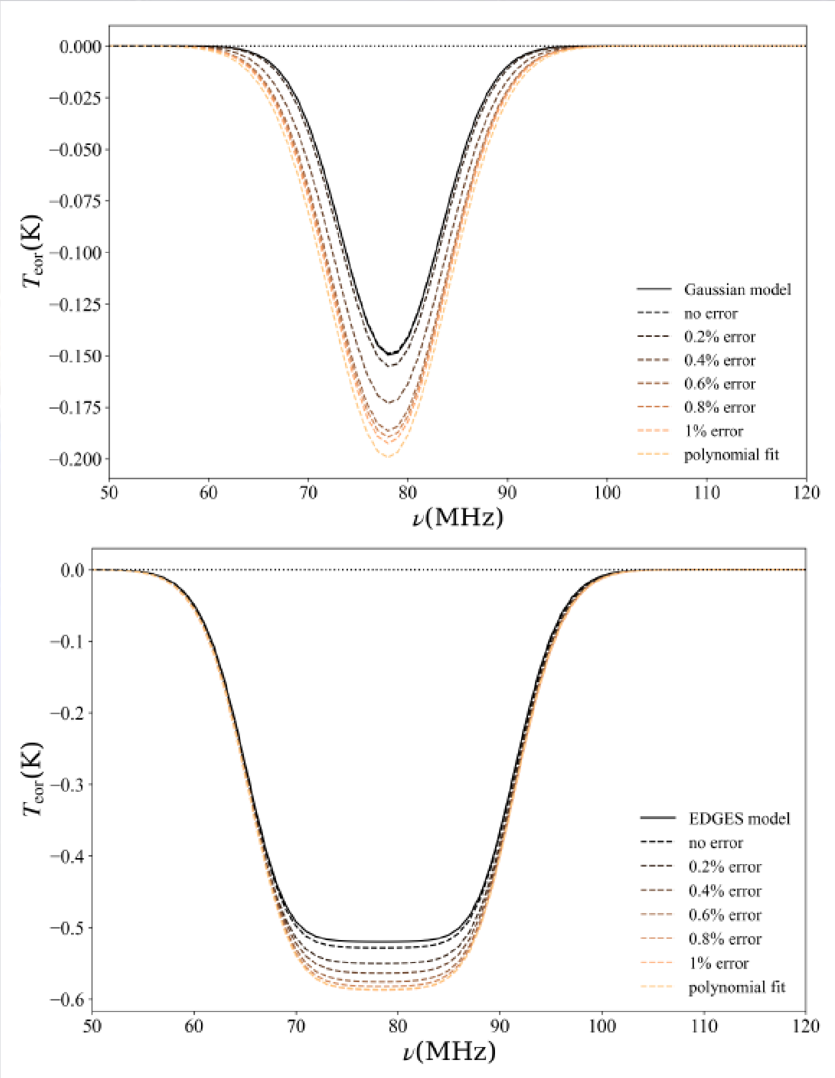
Inaccurate beam measurement

Completely random errors at both different frequencies and spatial positions

- VZOP will lost its advantage when error reaches 0.6%.
- VZOP will not be worse than common polynomial.



5. VZOP on DSL project

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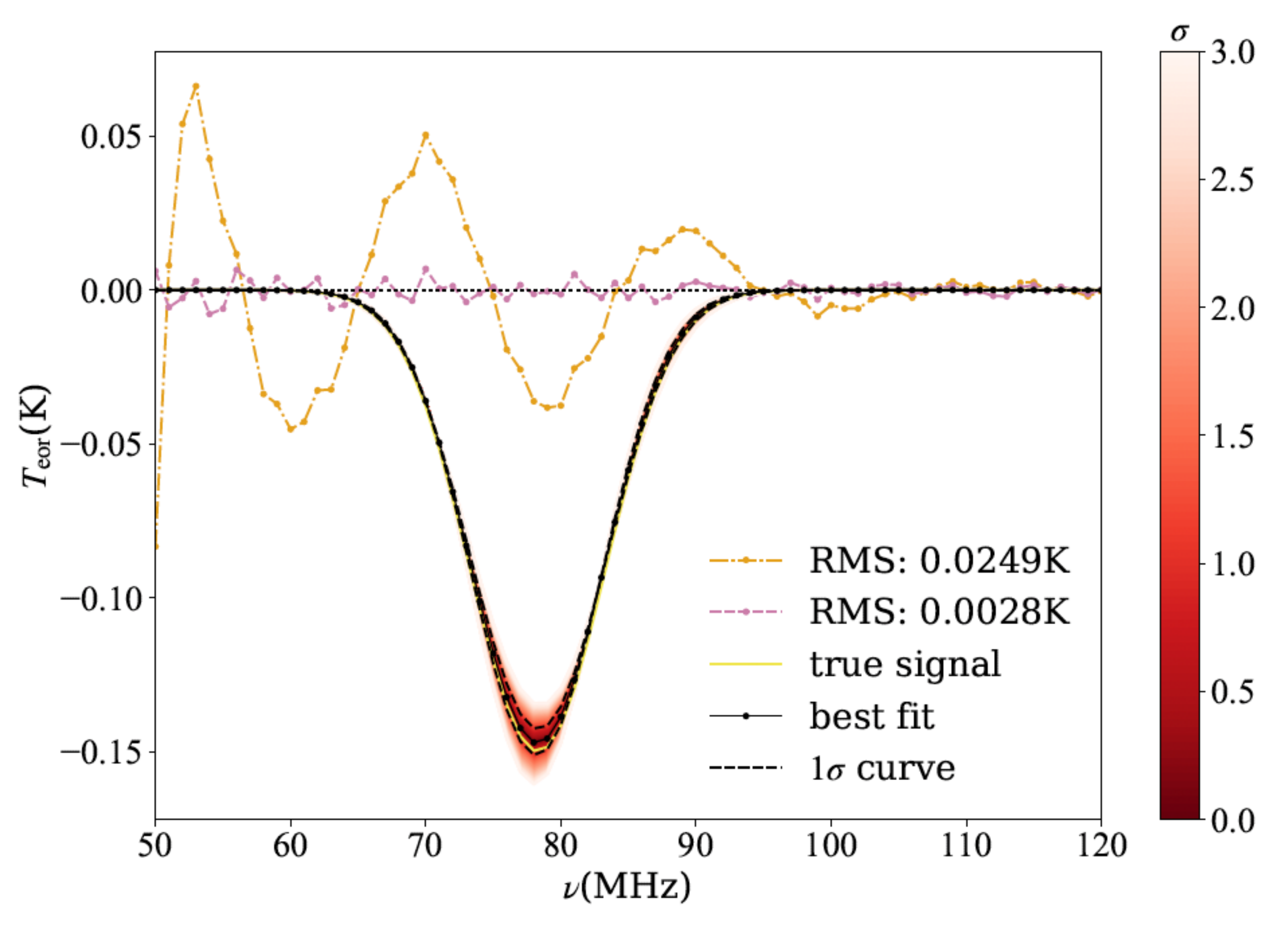
Considering the Lunar radiation



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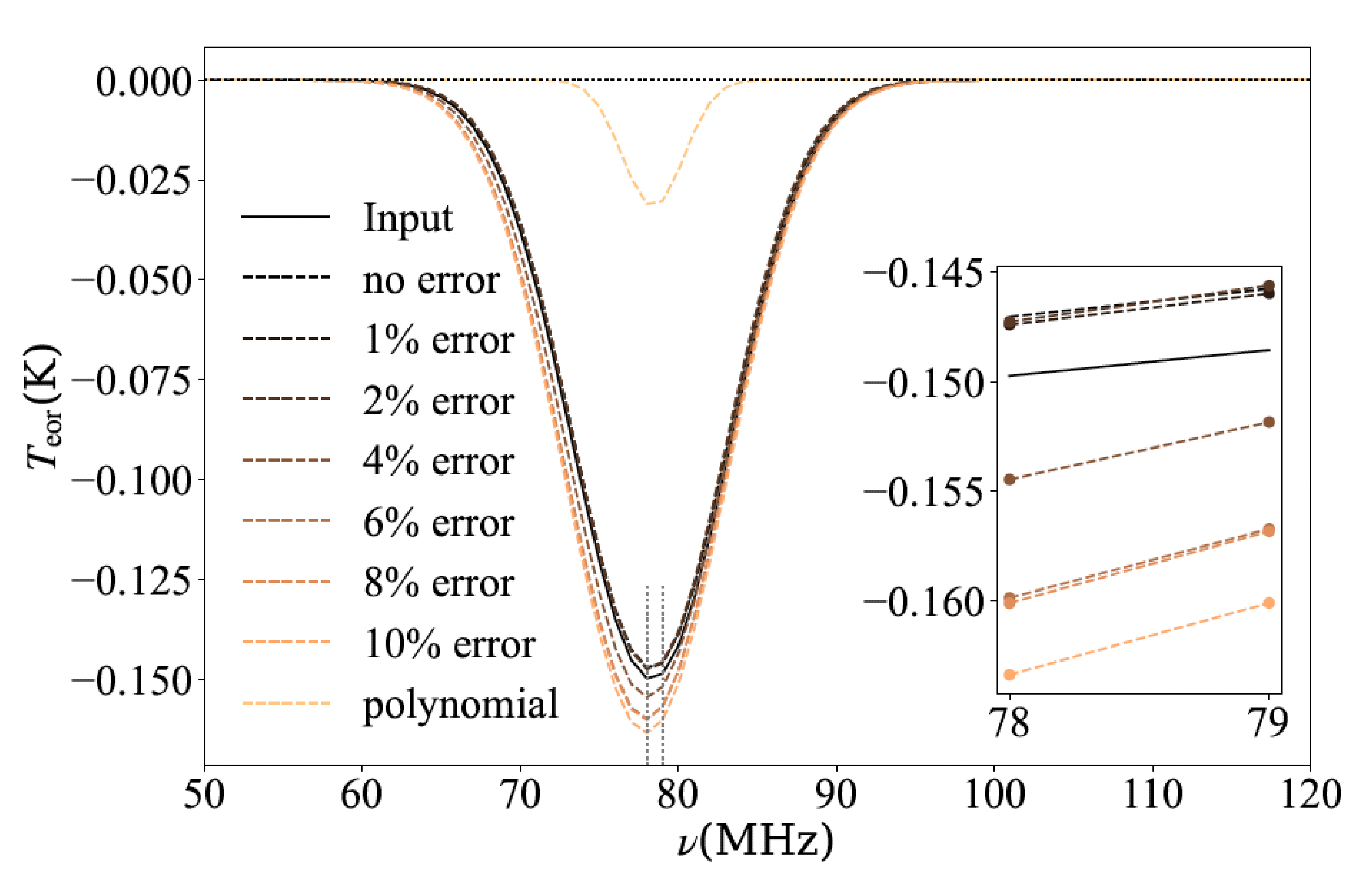
Inaccurate beam measurement



6. Conclusions

1. VZOP is a new method to correct the chromaticity that doesn’t need to simulate the sky map.

2. VZOP can accurately recover 21 cm signal even the errors exist.

3. No matter how large the errors are, the fitting results of VZOP will not be worse than common polynomial.

Thank You

If anyone is interested in my work, feel free to contact me for further discussion.

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