Precision Calibration Techniques for 21 cm Cosmology

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Tianlai Collaboration Meeting

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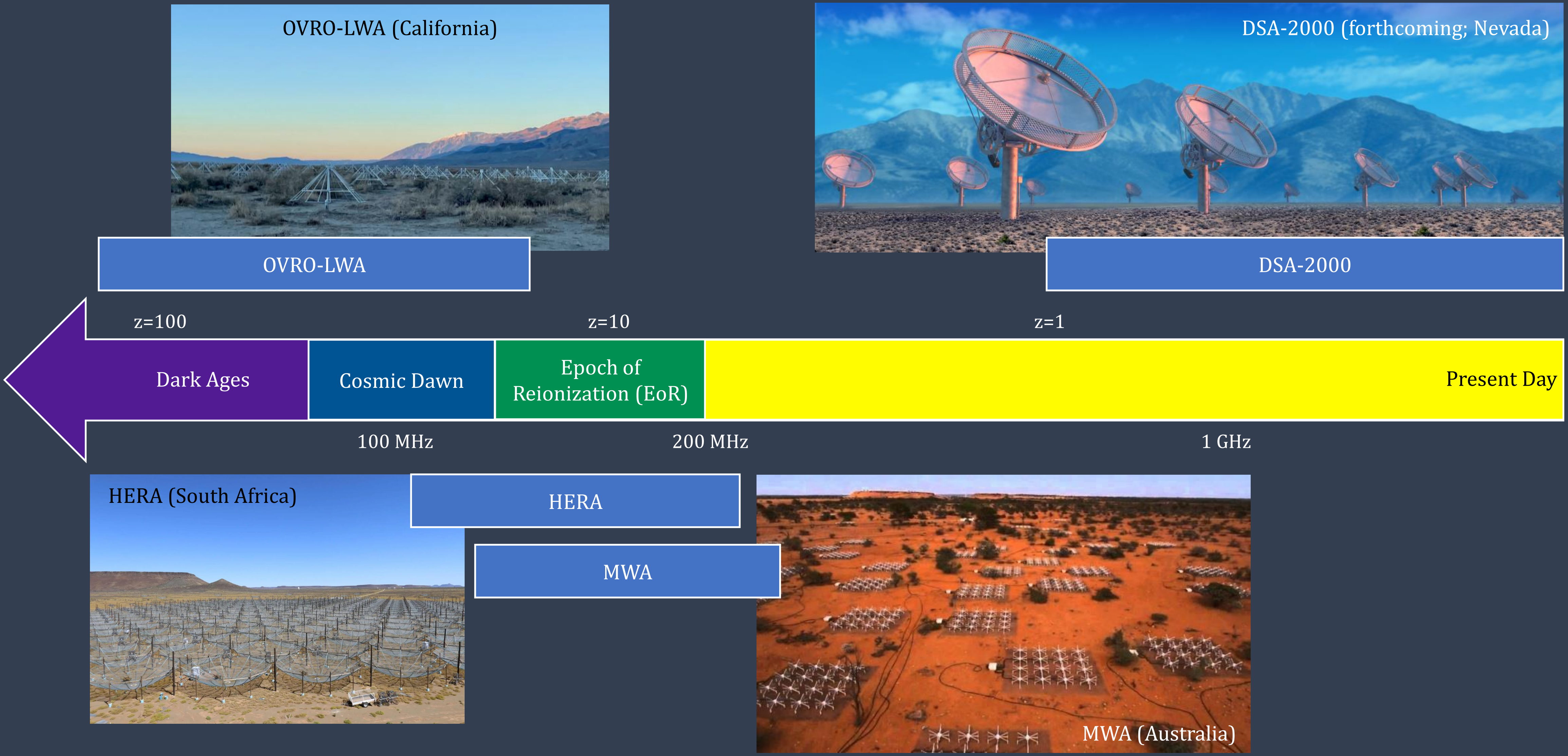
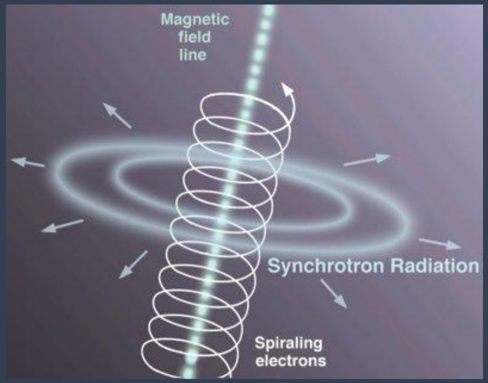
Caltech

21 cm Cosmology Across Redshift

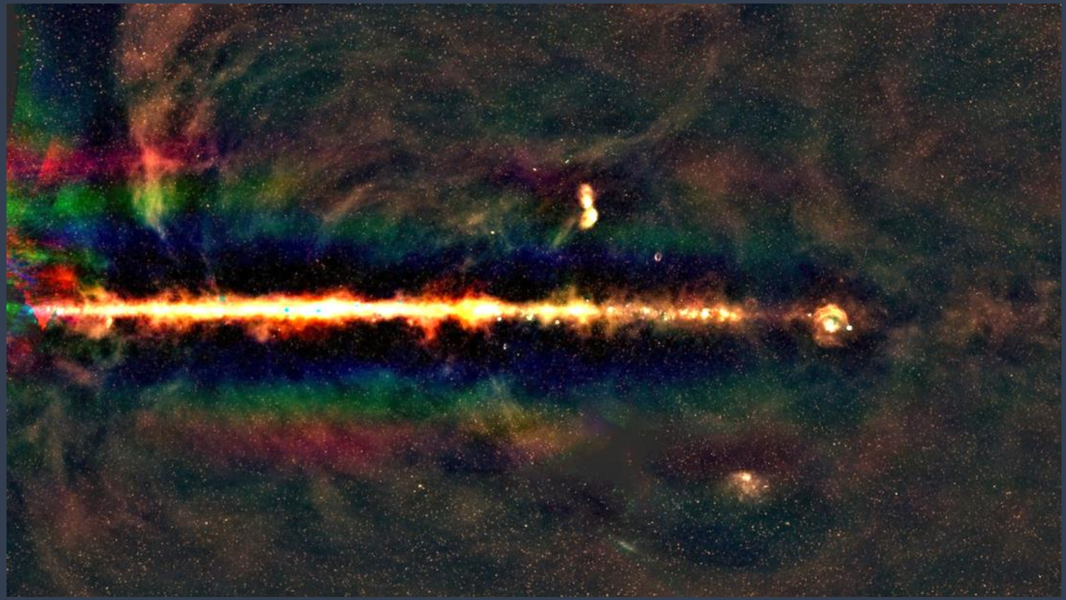


Foregrounds are the principal challenge for 21 cm cosmology



- "Foregrounds": everything between us and the high redshift signal
- Galactic emission from the Milky Way and other galaxies
- Synchrotron and bremsstrahlung (free-free) emission that is characteristically spectrally smooth
- 4-5 orders of magnitude brighter than the 21 cm signal
- Must be filtered from the data based on its spectral properties

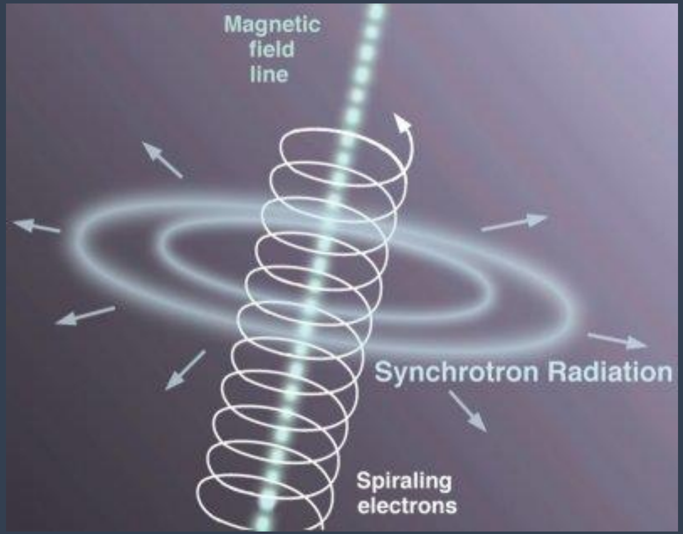
Astrophysical Foregrounds at 180 MHz, Measured by the MWA

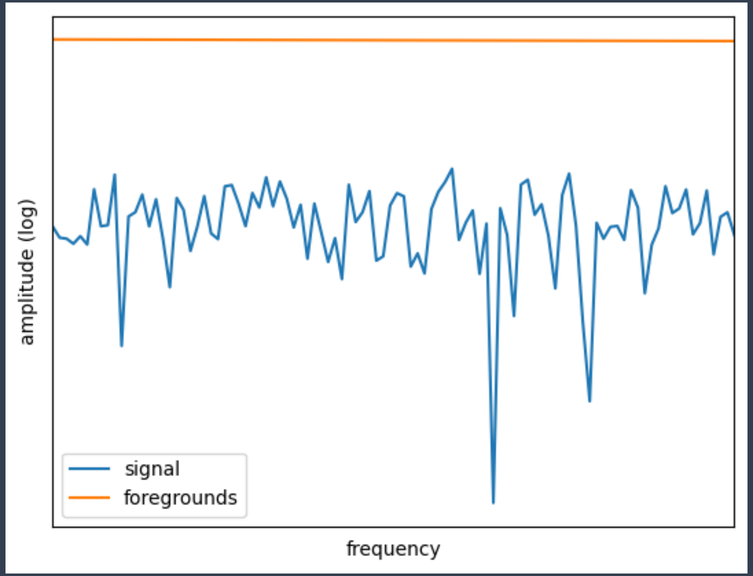


GLEAM collaboration, gleamoscope.icrar.org

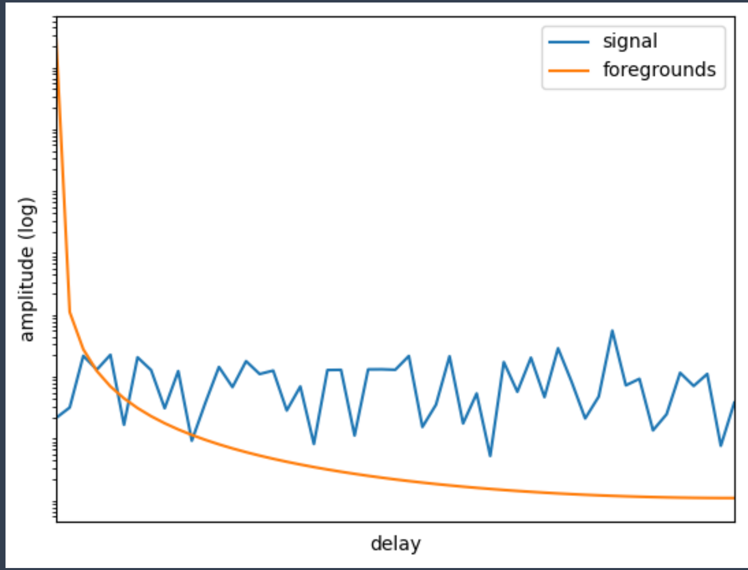
Foreground Emission

- Synchrotron and bremsstrahlung (free-free) emission
  - *Not* emission lines
- Inherently spectrally smooth
- Can be (in principle) filtered from the 21 cm signal based on its spectral properties

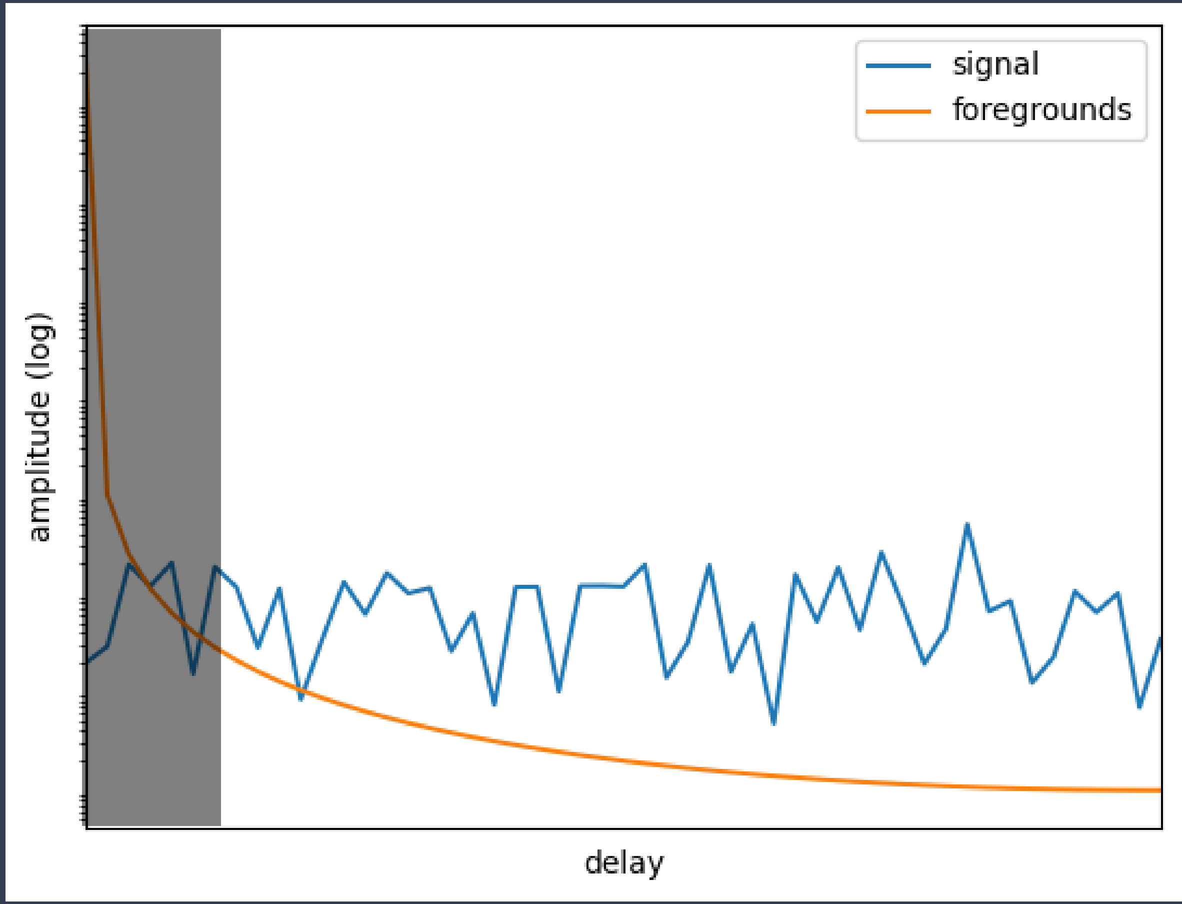




Fourier Transform

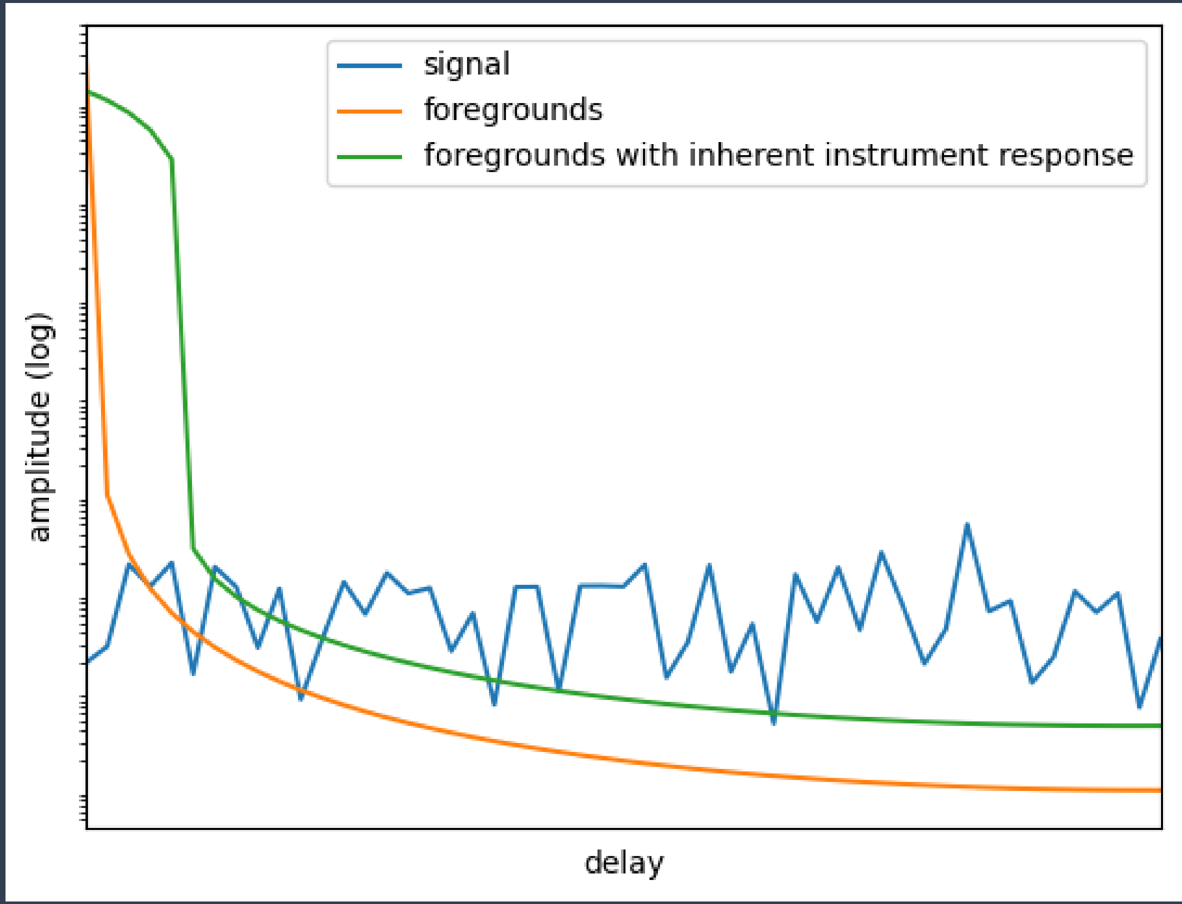


Foreground Filtering



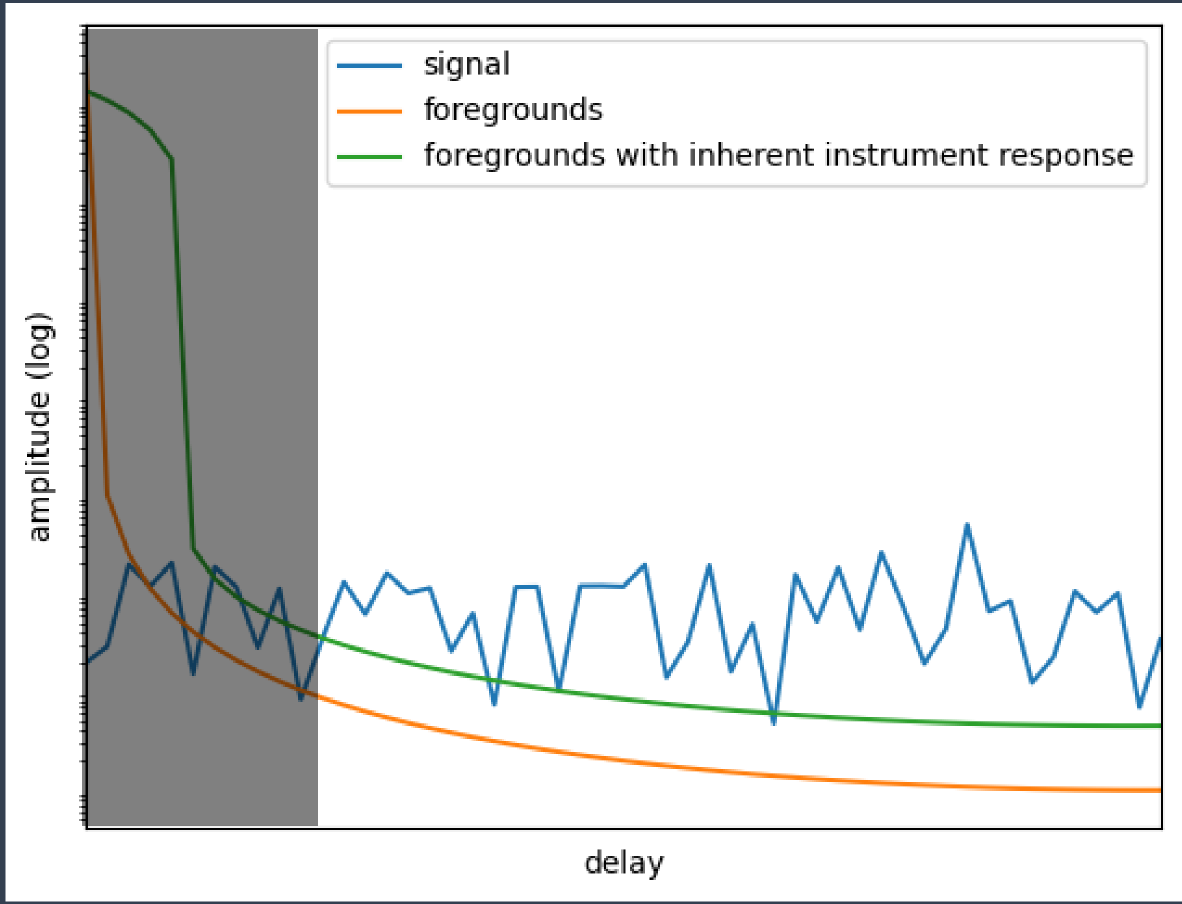
- Mask modes that are dominated by foregrounds
- Make the measurement on signal-dominated modes only

Foreground Filtering



- Mask modes that are dominated by foregrounds
- Make the measurement on signal-dominated modes only
- Instrumental effect extends foreground-contaminated modes

Foreground Filtering



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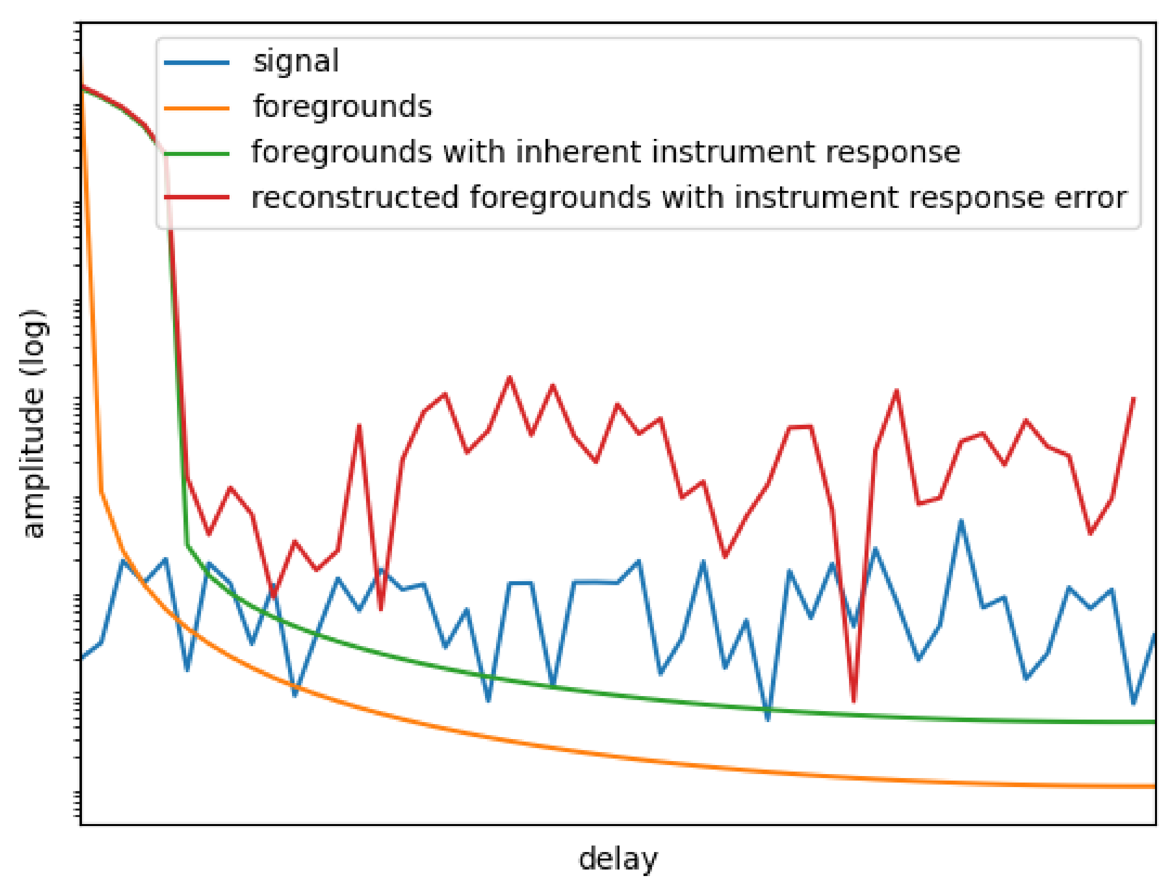
Foreground Filtering



- Mask modes that are dominated by foregrounds
- Make the measurement on signal-dominated modes only
- Instrumental effect extends foreground-contaminated modes
- Excess noise remains

Calibration error is the dominant source of systematic error for 21 cm cosmology analyses

What is calibration?

- Determining the instrument response to an incident sky signal
- Calculates the sensitivity and timing of each antenna
- Sensitivity and timing may vary as a function of frequency and time

Two Classes of Calibration

Direction-Independent Calibration

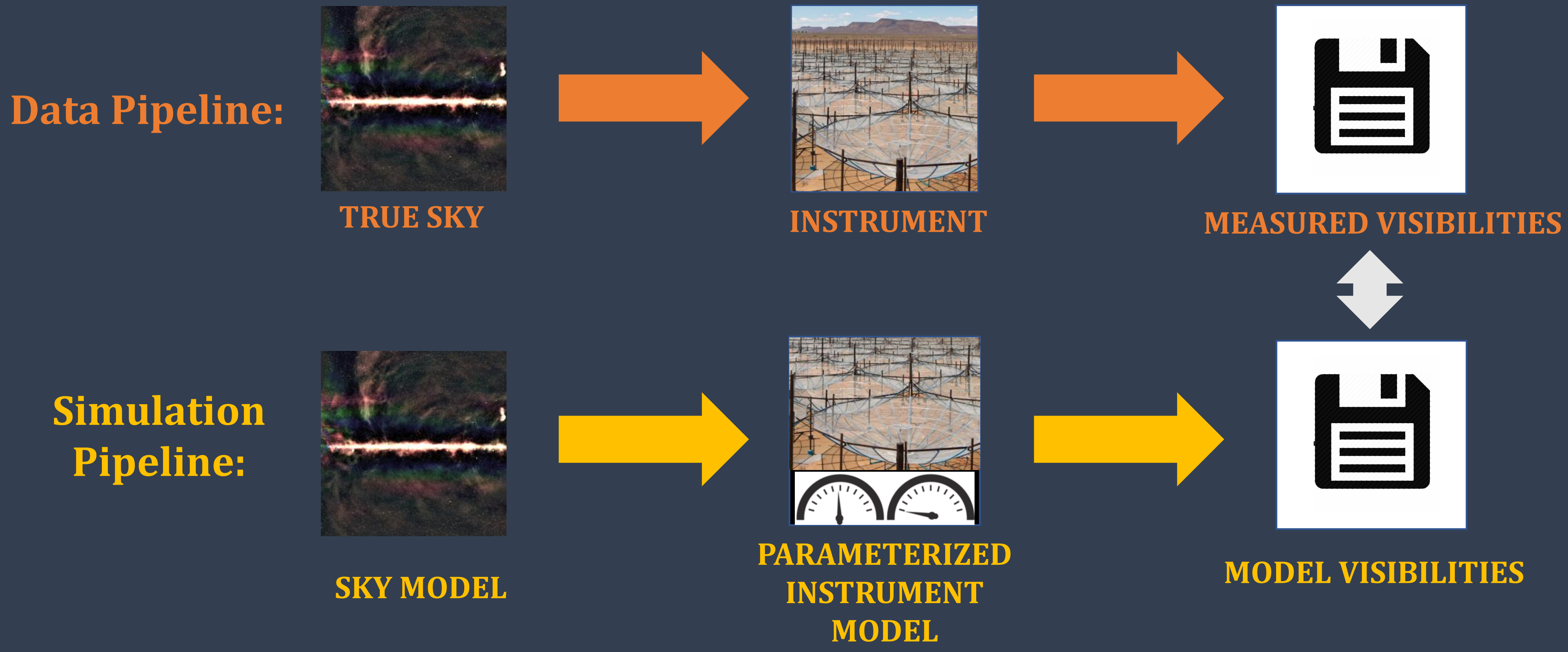
- Fits a gain per antenna, frequency, and time interval
- Does not account for direction-independent (beam) effects

Direction-Dependent Calibration

- Calculates the direction-dependent beam response
- Fits several or many parameters per antenna

This presentation focuses on direction-independent calibration

Antenna gains are calculated empirically by matching data to expectation values



The Measurement Equation

$$v_{ab} = g_a g_b^* u_{ab} + n_{ab}$$

measured visibility

antenna gains

true visibility

noise

Calibration Frameworks

1. **Sky-Based Calibration**
2. Redundant Calibration
3. Unified Calibration
4. Delay-Weighted Calibration

Sky-Based Calibration

- Standard radio interferometric calibration
- Sky is modeled, and the simulation is assumed to be a good approximation of the true signal

$$v_{ab} = g_a g_b^* \overset{m_{ab}}{\cancel{u_{ab}}} + n_{ab}$$

- Fit the gains with a least-squares minimization at each frequency

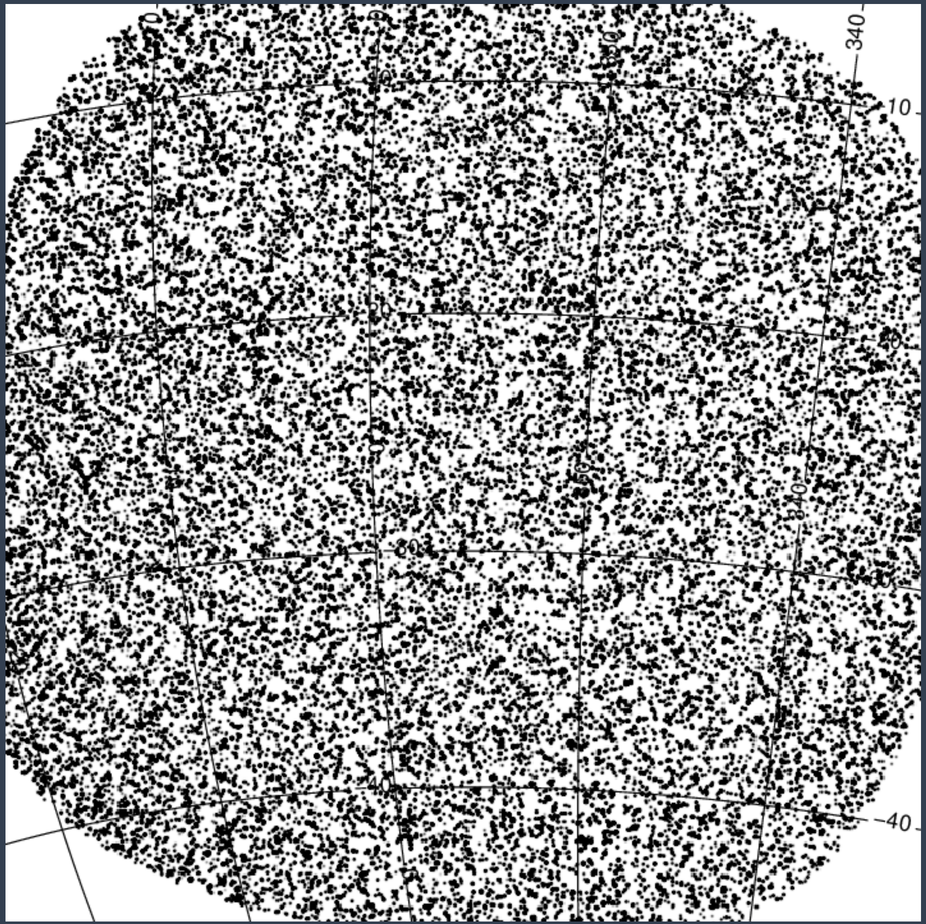
$$\chi^2(\vec{g}) = \sum_{ab} |v_{ab} - g_a g_b^* m_{ab}|^2$$

What is the impact of sky model errors on calibration?

Simulation framework:

1. Simulate data from a detailed sky model
2. Perturb the sky model
   - Omit the faintest sources (< 100 mJy)
3. Calibrate the simulated data to the perturbed sky model
4. Compare the original simulated data to the calibrated simulated data

GLEAM catalog, as used for MWA calibration

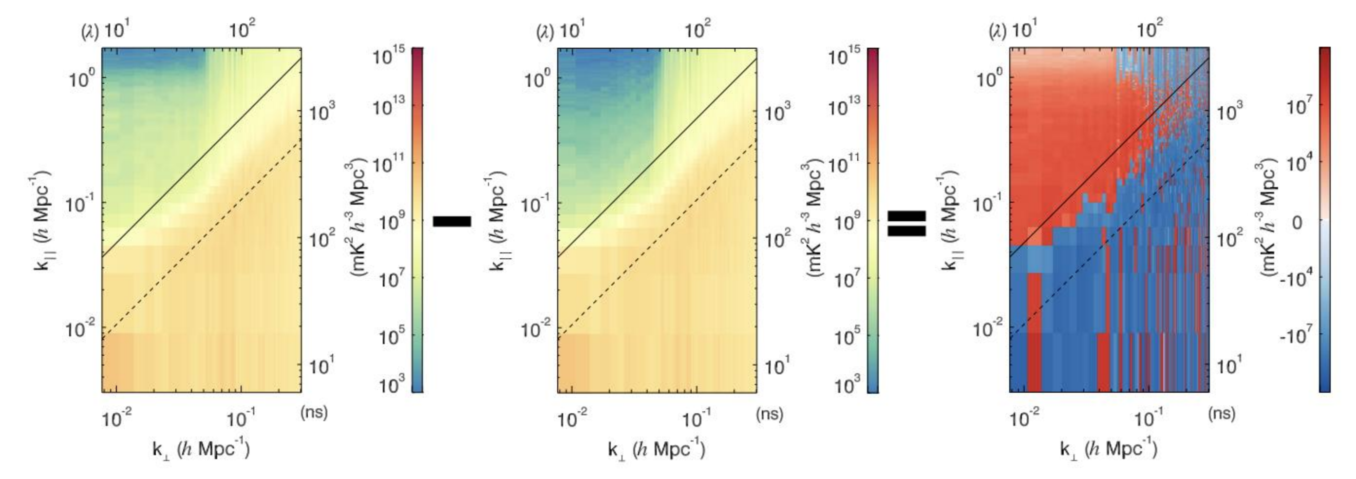


Sky-based calibration is not precise enough for 21 cm cosmology

- Low-level errors in the sky model used to model visibilities produce calibration errors that swamp the signal

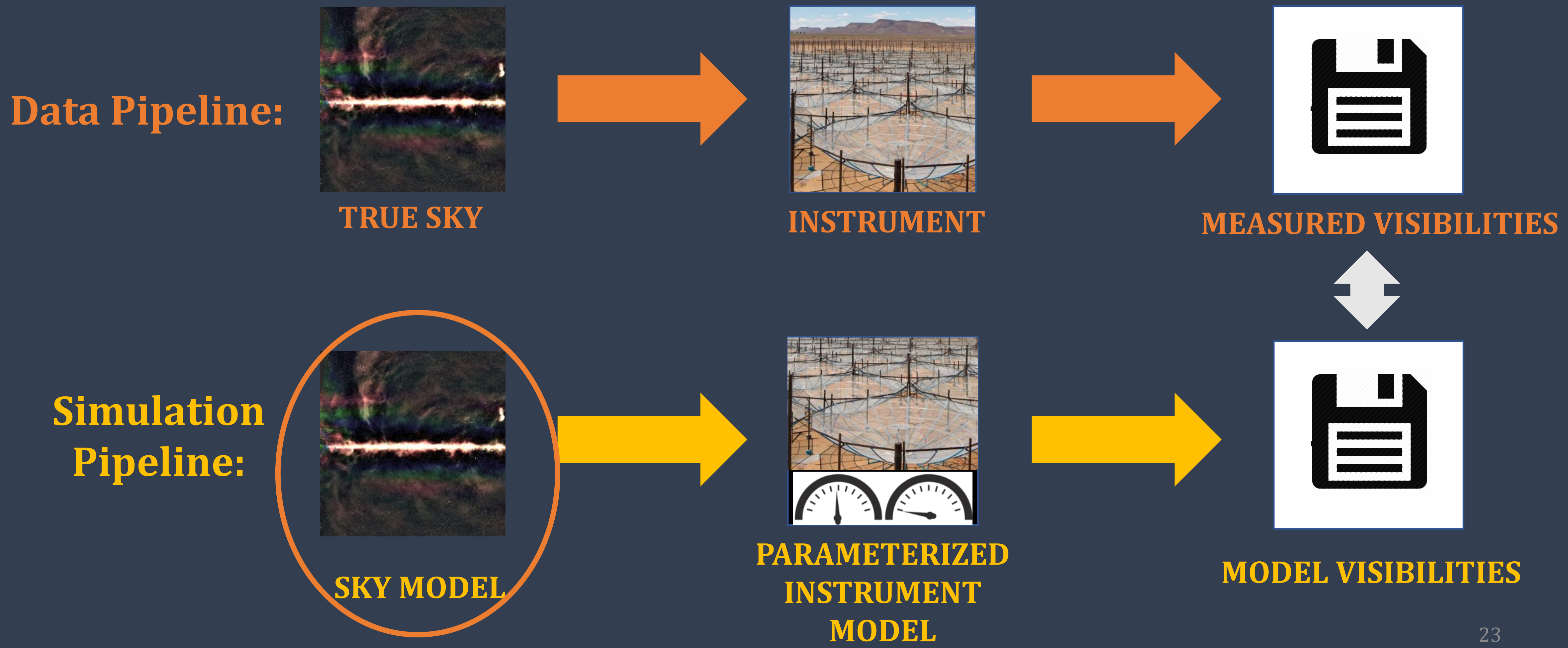
Simulation of MWA data, calibrated with low-level sky model error

Simulation of MWA data, perfect calibration

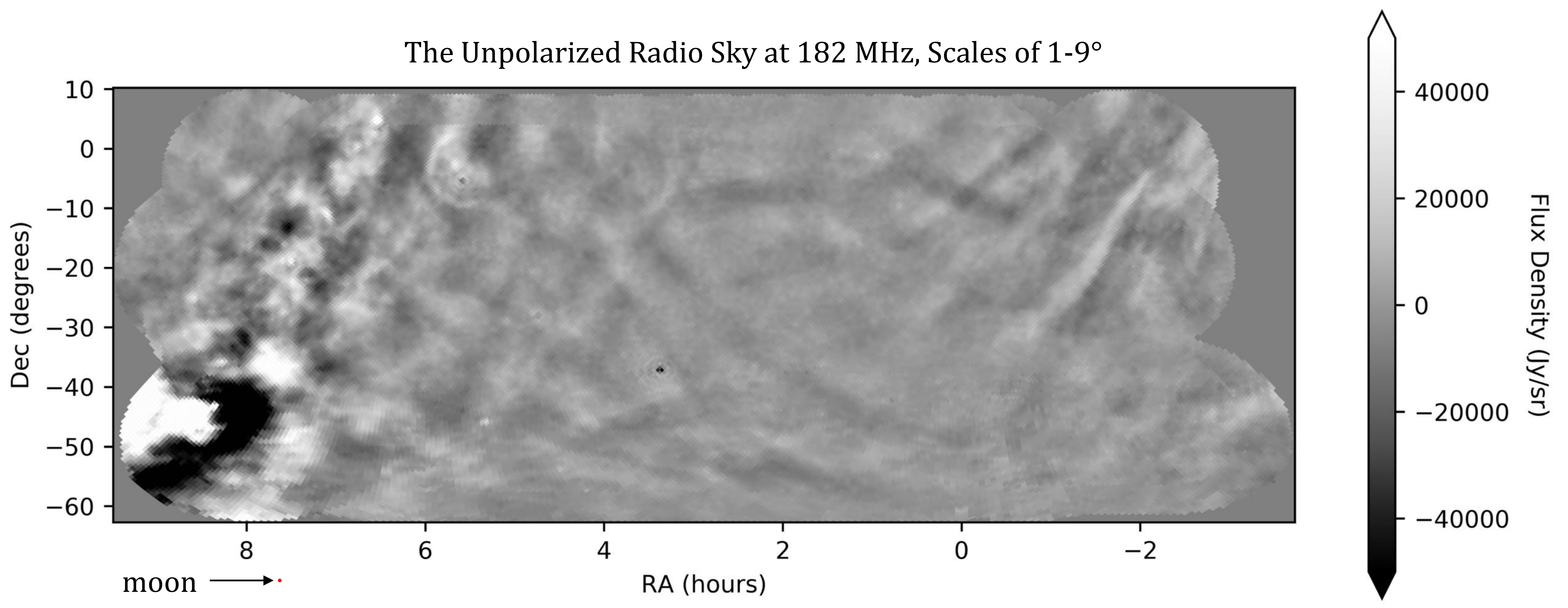


Barry+ 2016

We need a better sky model!

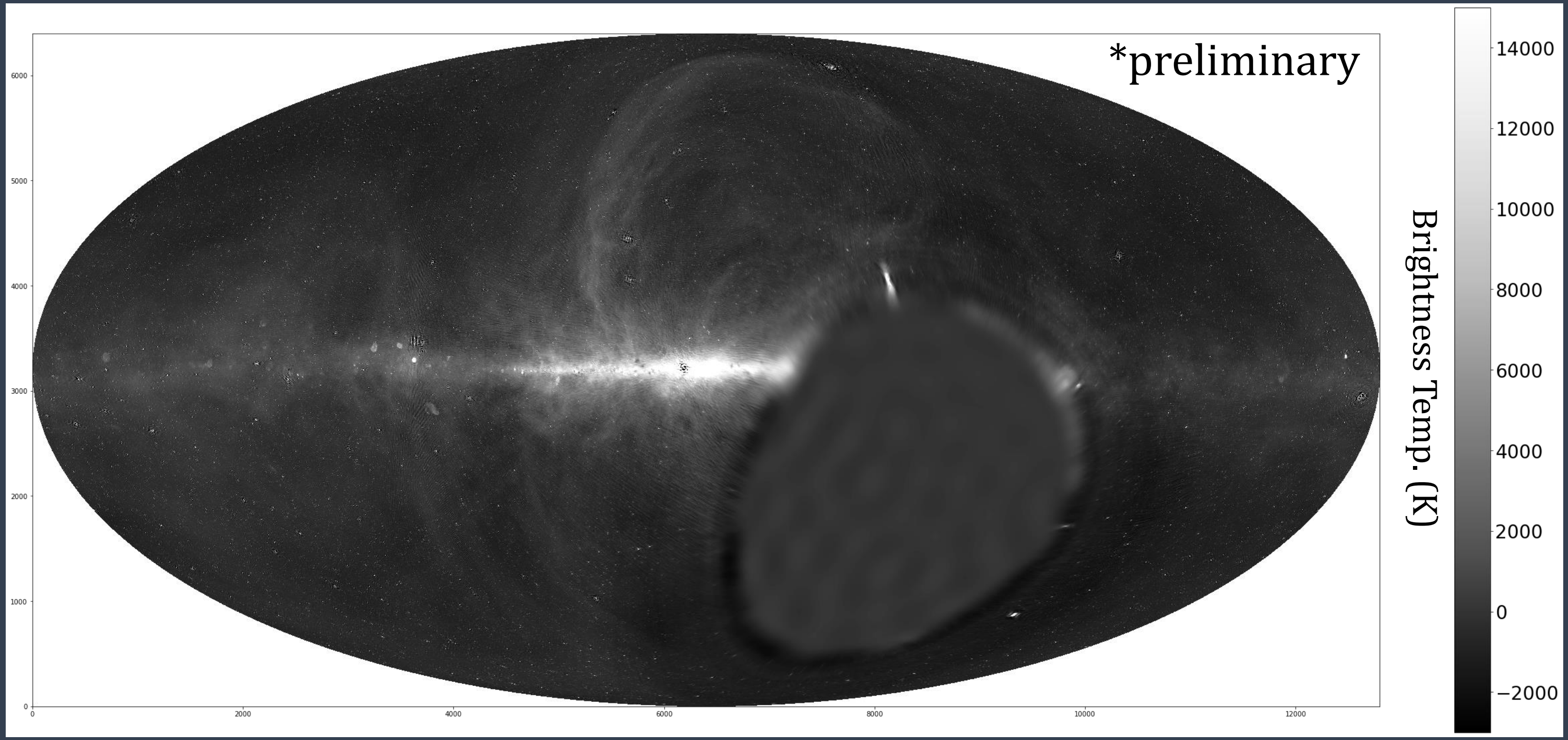


Mapping Large-Scale Diffuse Structure at 182 MHz with the MWA



Byrne+ 2021b

All-Sky Mapping at 73 MHz with the OVRO-LWA

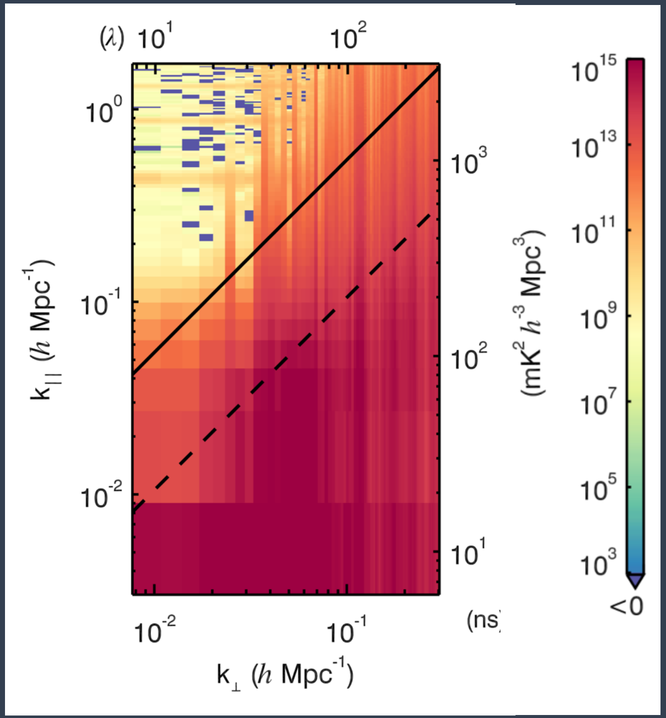


Xander Hall

- Diffuse mapping for 21 cm cosmology and other applications
- Uses m-mode analysis: all-sky image reconstruction with at least 24 hours of data

Better sky models work! *...somewhat...*

MWA data, calibrated to an incomplete sky model with compact sources only



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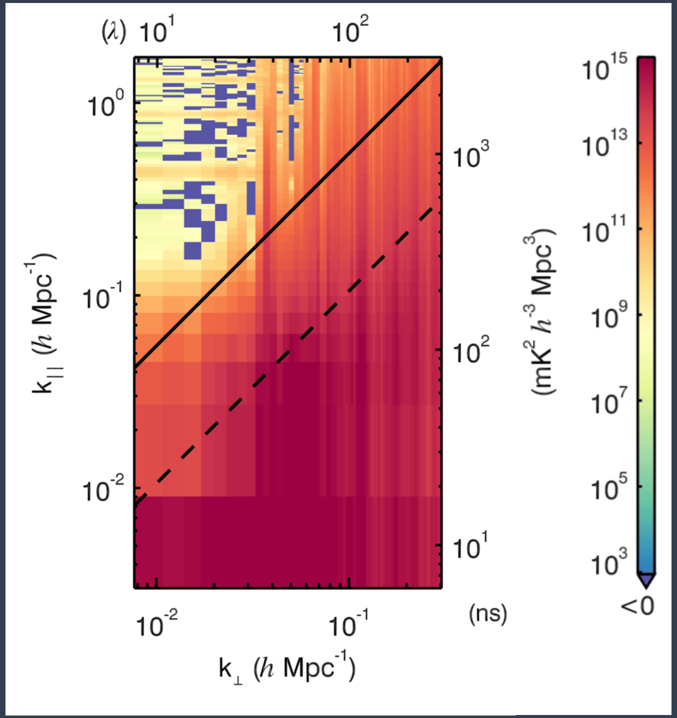
MWA data, calibrated to a sky model that includes diffuse emission



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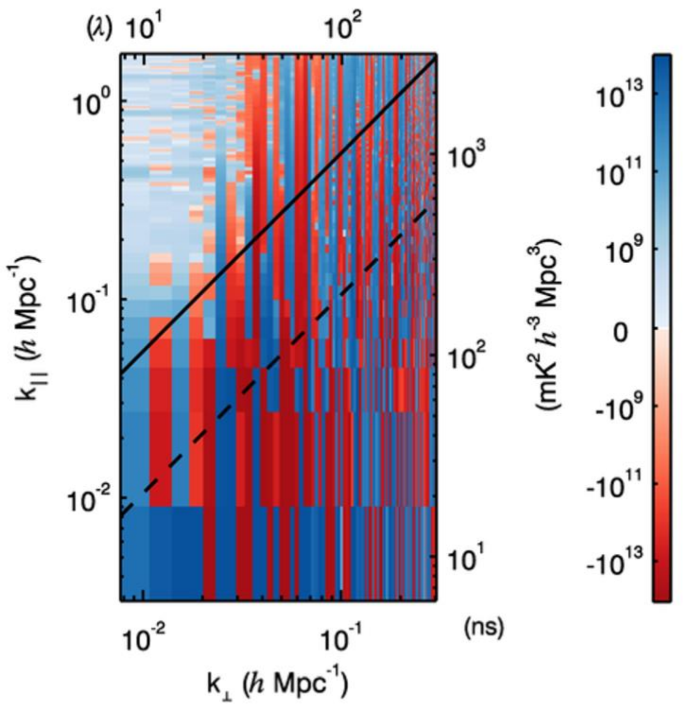
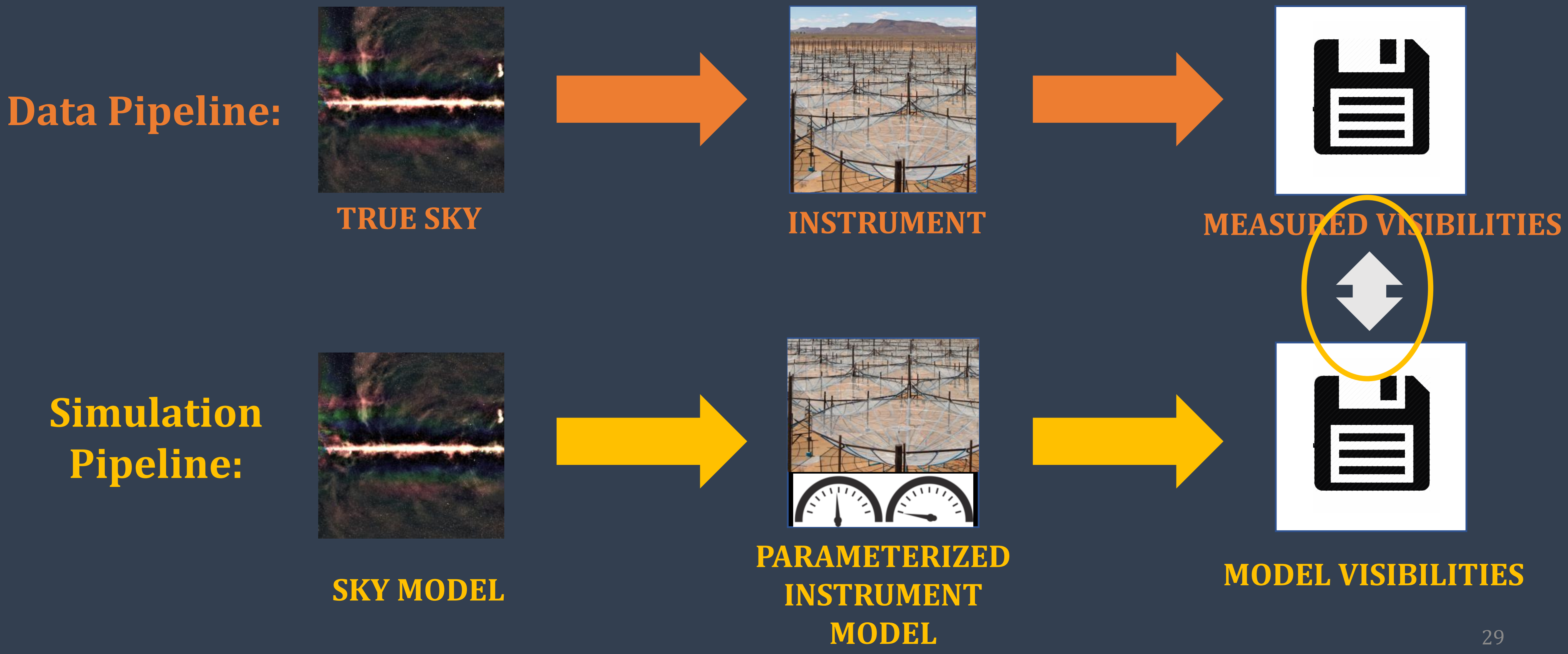


We need more advanced calibration approaches that are resilient to error.

We need more statistically robust calibration

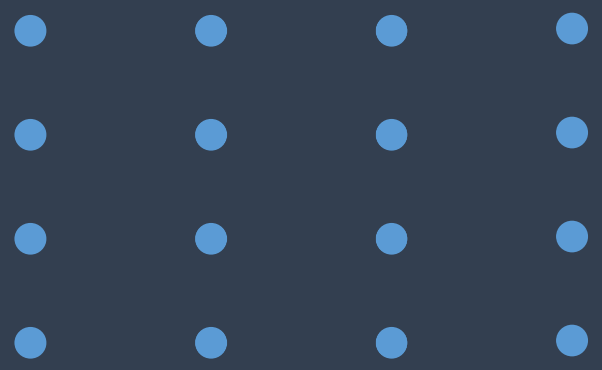


Calibration Frameworks

1. Sky-Based Calibration
2. **Redundant Calibration**
3. Unified Calibration
4. Delay-Weighted Calibration

Redundant Calibration

- Works only for regular arrays



Redundant Calibration

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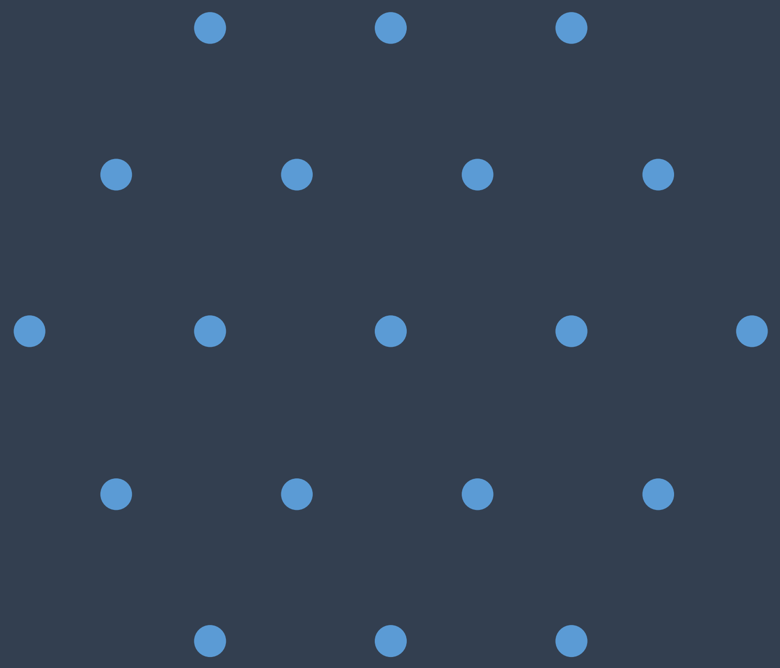
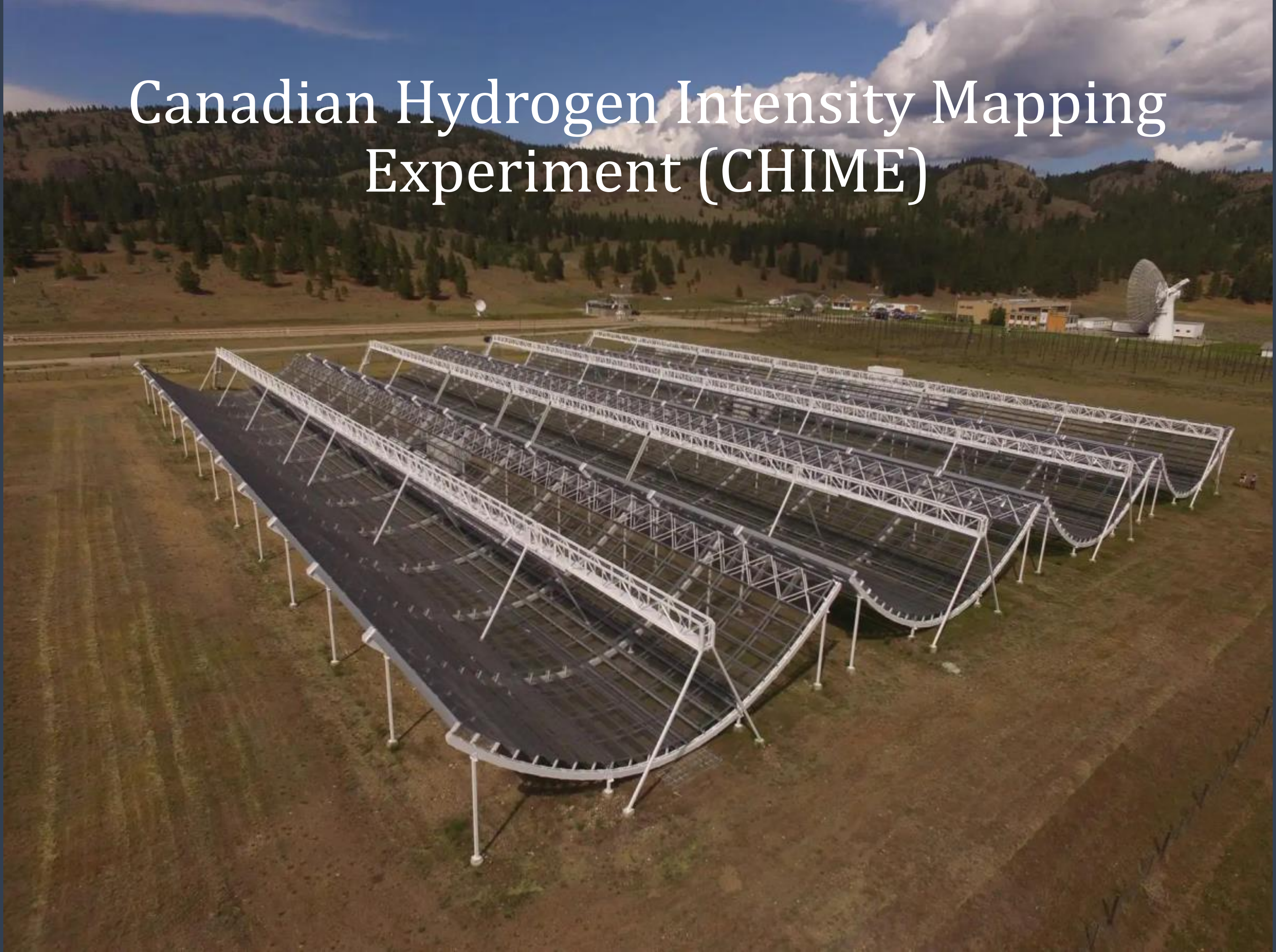


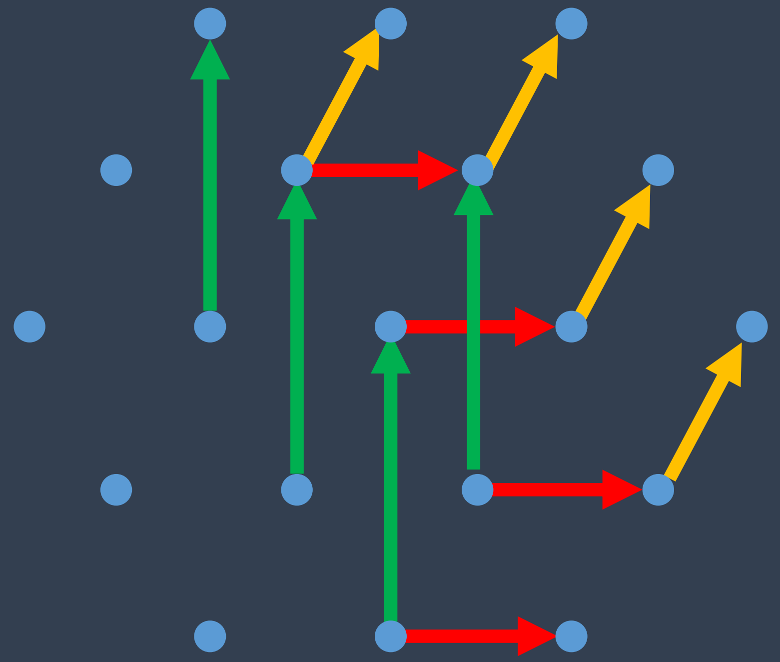
Hydrogen Epoch of Reionization Array (HERA)

Canadian Hydrogen Intensity Mapping Experiment (CHIME)



Redundant Calibration

- Works only for regular arrays
- Calibrates the relative antenna response by matching repeated baseline measurements



Redundant Calibration

- Works only for regular arrays
- Calibrates the relative antenna response by matching repeated baseline measurements
- j indicates the redundant baseline set

$$v_{ab} = g_a g_b^* \overset{u_j}{\cancel{u_{ab}}} + n_{ab}$$

$$\chi^2(\vec{g}, \vec{u}) = \sum_j \sum_{\{ab\} \in j} \left| v_{ab} - g_a g_b^* u_j \right|^2$$

What are the (implicit) assumptions of redundant calibration?

$$\chi^2(\vec{g}, \vec{u}) = \sum_j \sum_{\{ab\} \in j} \left| v_{ab} - g_a g_b^* u_j \right|^2$$

- Visibilities within a redundant baseline set are identical up to the noise
- Enforcing redundancy is more trustworthy than sky modeling

Redundant calibration solutions are *degenerate*

$$\chi^2(\vec{g}) = \sum_j \sum_{\{ab\}\in j} \left|v_{ab} - g_a g_b^* u_j\right|^2$$

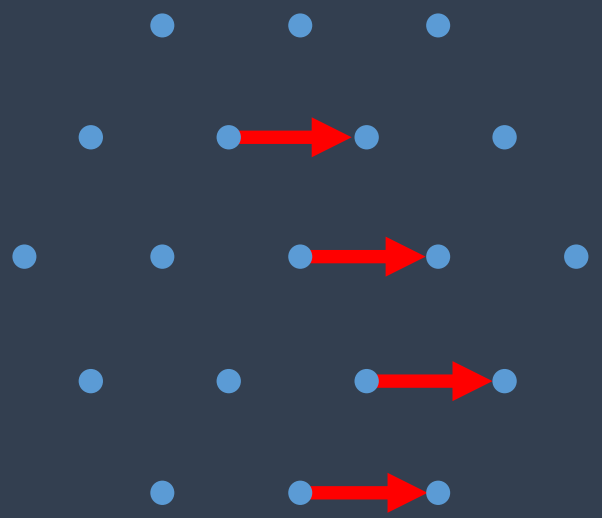
- χ^2 is unchanged under certain transformations:
  - Amplitude degeneracy: $g_a \to A g_a, u_j \to \frac{1}{A^2} u_j$
  - Phase degeneracy: $g_a = |g_a|e^{i\phi_i} \to |g_a|e^{i(\phi_i+\Delta)}$
  - Phase gradient degeneracy: $g_a = |g_a|e^{i\phi_a} \to |g_a|e^{i\left(\phi_a+\Delta_x x_a+\Delta_y y_a\right)}$

$$u_j = \left|u_j\right|e^{i\phi_j} \to \left|u_j\right|e^{i\left(\phi_j+\Delta_x(x_a-x_b)+\Delta_y(y_a-y_b)\right)}$$

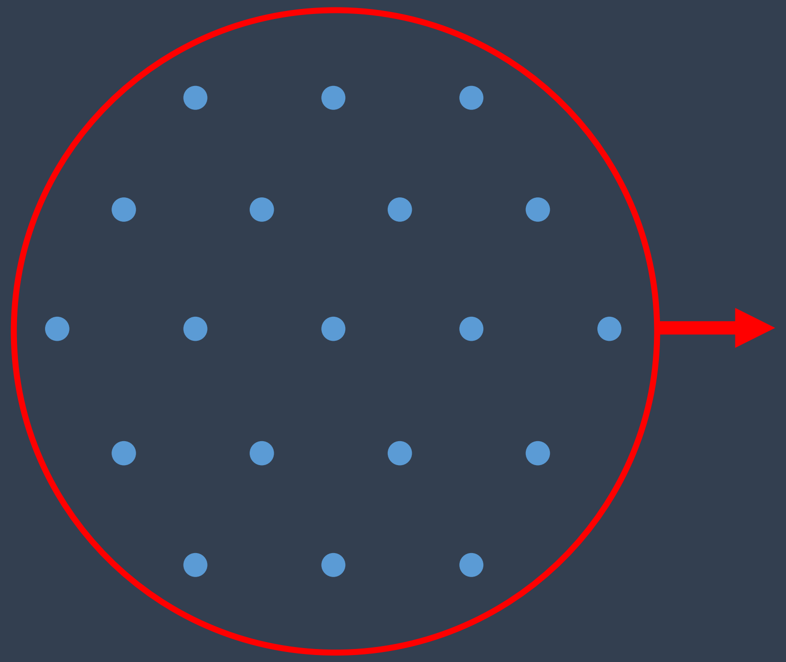
- Degenerate parameters correspond to the bulk array response
- 4 degenerate parameters per frequency

Redundant calibration consists of two steps

1. Relative calibration
    - Use redundancy
    - Fits $(2 \times N_{ants} - 4) \times N_{freqs}$ parameters



2. Absolute calibration
    - Constrain degeneracies from a sky model
    - Fits $4 \times N_{freqs}$ parameters

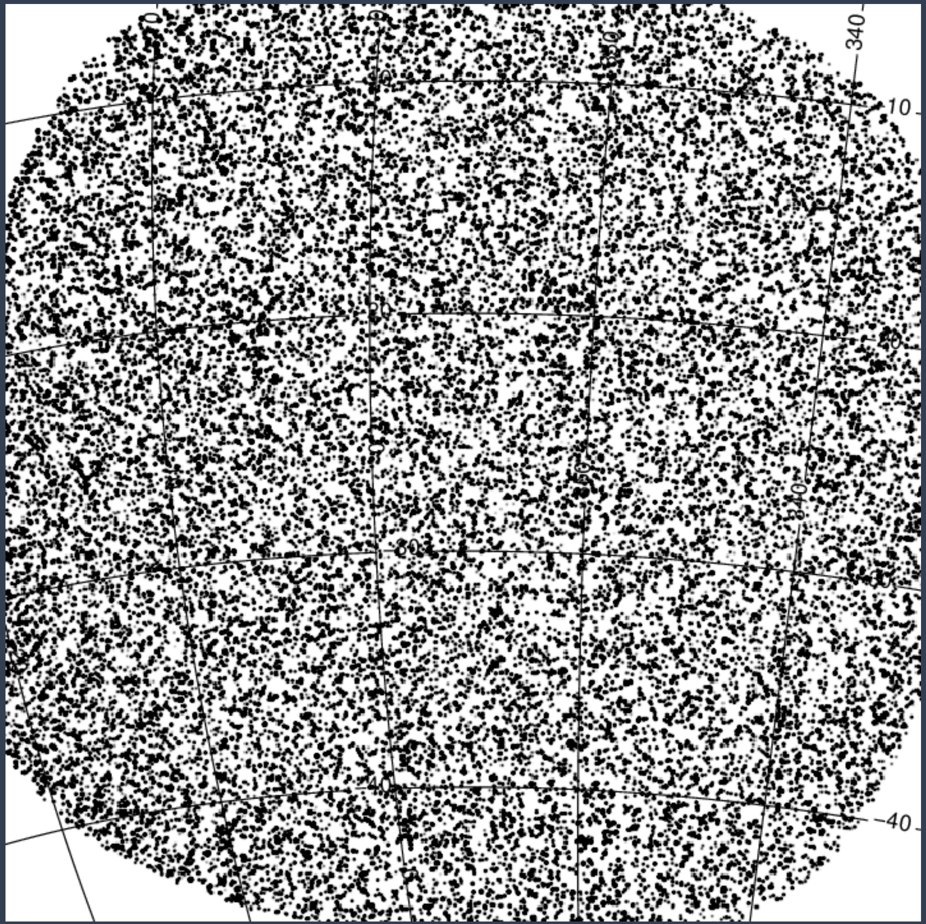


What is the impact of sky model errors on redundant calibration?

Simulation framework:

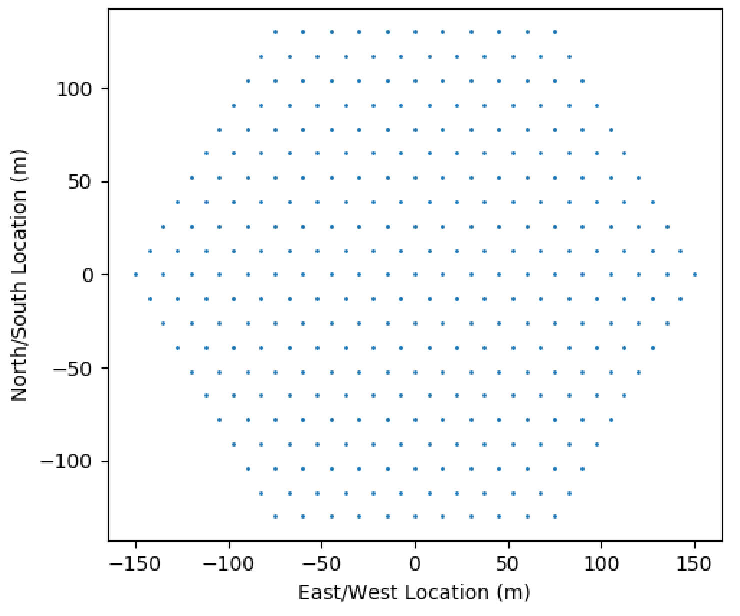
1. Simulate data from a detailed sky model
2. Perturb the sky model
   - Omit the faintest sources
3. Perform absolute calibration on the simulated data, using the perturbed sky model
   - Hold relative calibration constant
4. Compare the original simulated data to the calibrated simulated data

GLEAM catalog, as used for MWA calibration

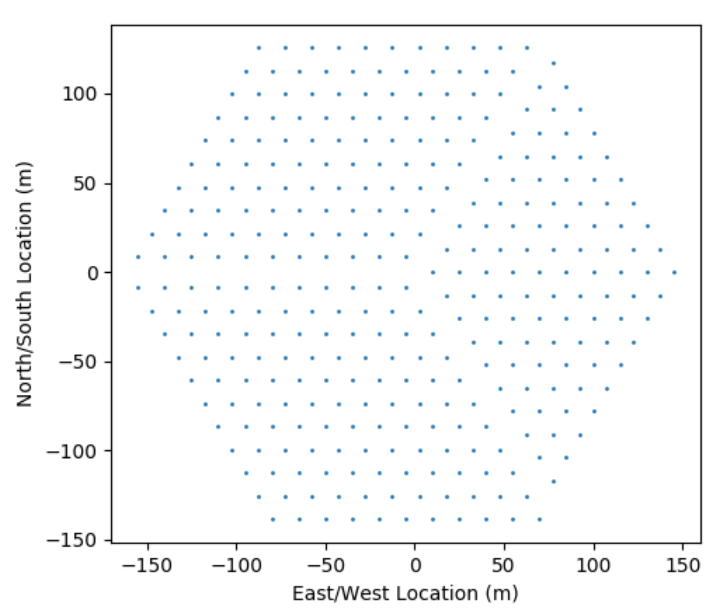


Simulated Arrays

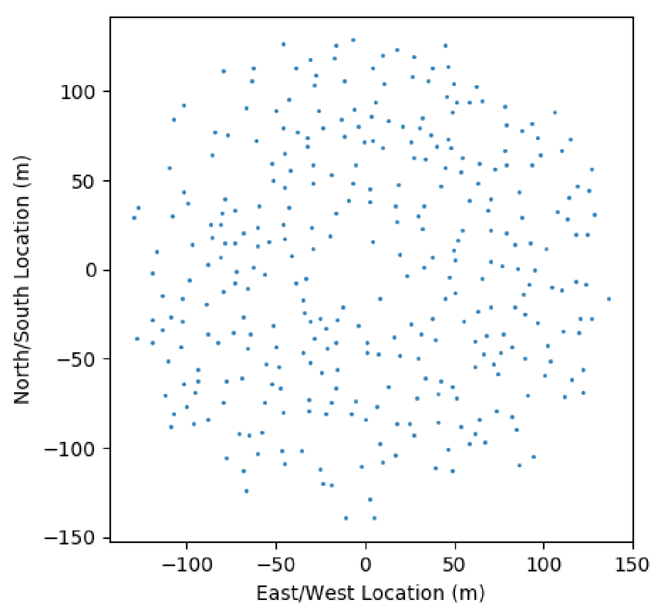
Hexagonal



Offset Hexagonal



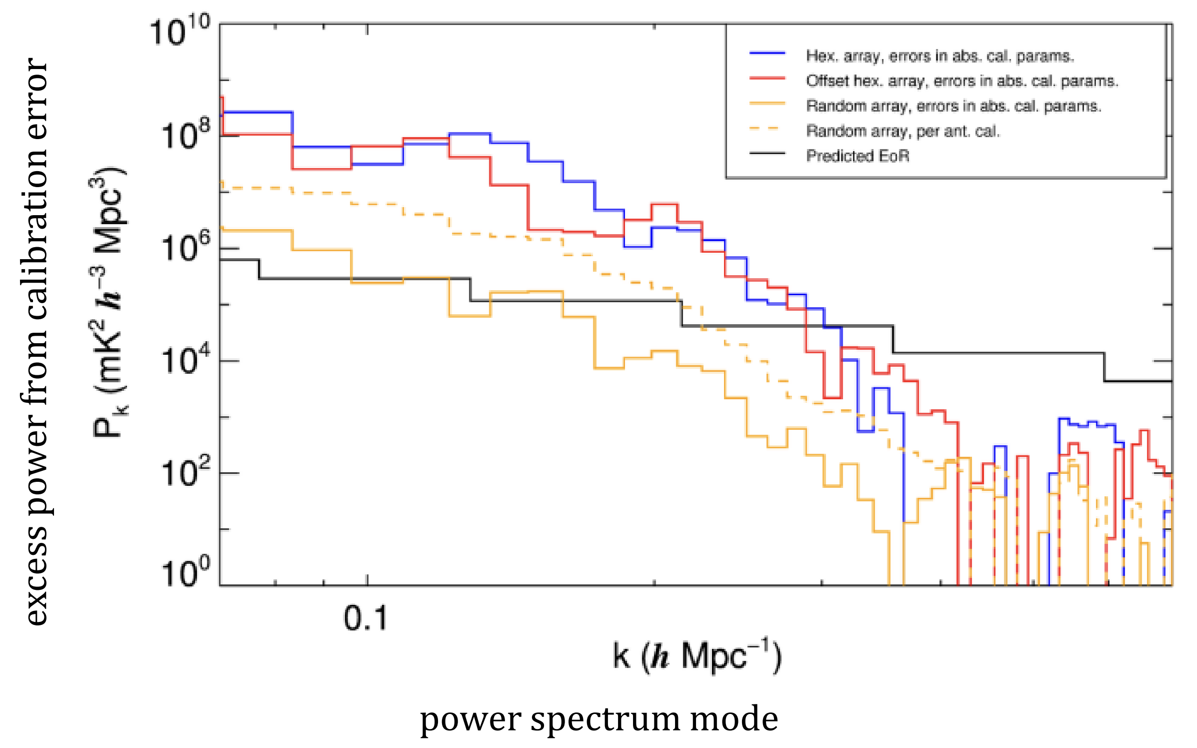
Random



Redundant calibration is not precise enough for 21 cm cosmology

- Low-level errors in the sky model produce calibration errors that swamp the signal
- The effect is worse for regular arrays
  - Regular arrays have fewer independent visibility measurements that can constrain calibration

Simulated Calibration Error
with Low-Level Sky Model Error



Byrne+ 2019

Key Takeaways: Redundant Calibration and Sky Model Error (Byrne+ 2019)

1. Redundant calibration, like sky-based calibration, is highly sensitive to sky model error
2. Redundantly calibrated regular arrays are *more* sensitive to sky model error than randomized arrays calibrated with sky-based calibration
3. Future arrays built for systematics resilience should be randomized
4. We require new precision calibration techniques

Calibration Frameworks

1. Sky-Based Calibration
2. Redundant Calibration
3. **Unified Calibration**
4. Delay-Weighted Calibration

Unified Calibration (Byrne+ 2021a)

- Variant of redundant calibration that incorporates Bayesian priors on the fit visibilities

Bayes' Theorem

$$P(model \mid data) \propto P(data \mid model)\, P(model)$$

<u>likelihood function</u>
calibration consists of
maximizing this

<u>data distribution</u>
describes the
thermal noise

<u>prior</u>

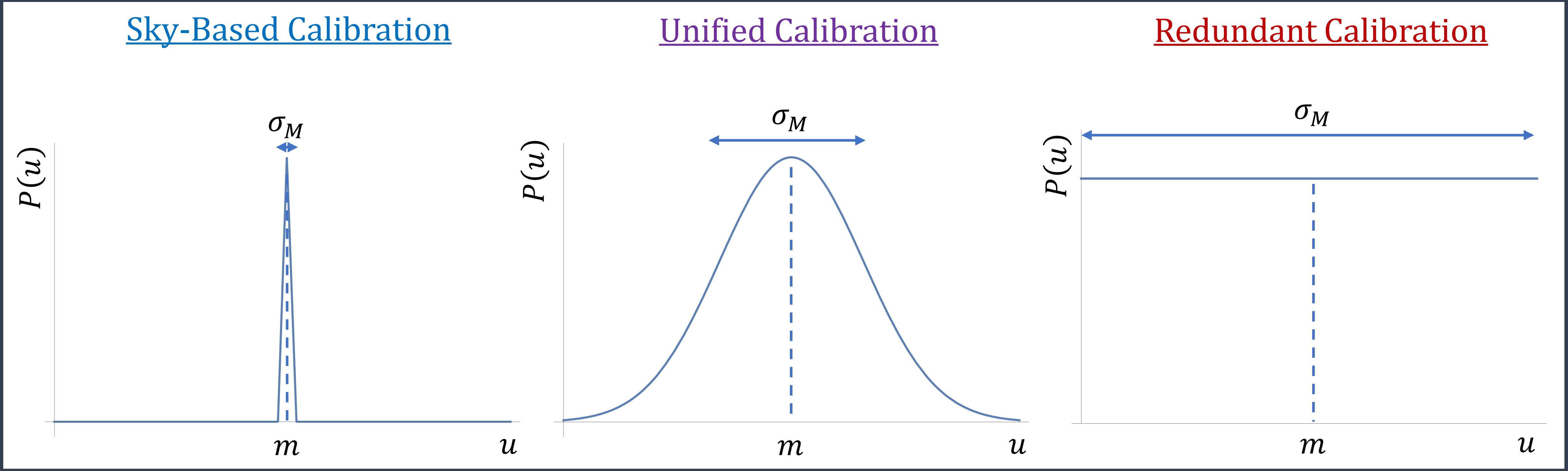
Unified Calibration (Byrne+ 2021a)

- Variant of redundant calibration that incorporates Bayesian priors on the fit visibilities

$$\chi^2(\vec{g},\vec{u}) = \underbrace{\frac{-1}{2\sigma_{\mathrm{T}}^2}\sum_{j}\sum_{\{ab\}\in j}\left|v_{ab} - g_a g_b^* u_j\right|^2}_{\text{redundant calibration}} - \underbrace{\frac{1}{2\sigma_{\mathrm{M}}^2}(\vec{u}-\vec{m})^\dagger \mathbf{C}_{\mathrm{R}}(\vec{u}-\vec{m})}_{\text{prior}}$$

- Operates midway between sky-based and redundant calibration

The Unified Calibration Prior



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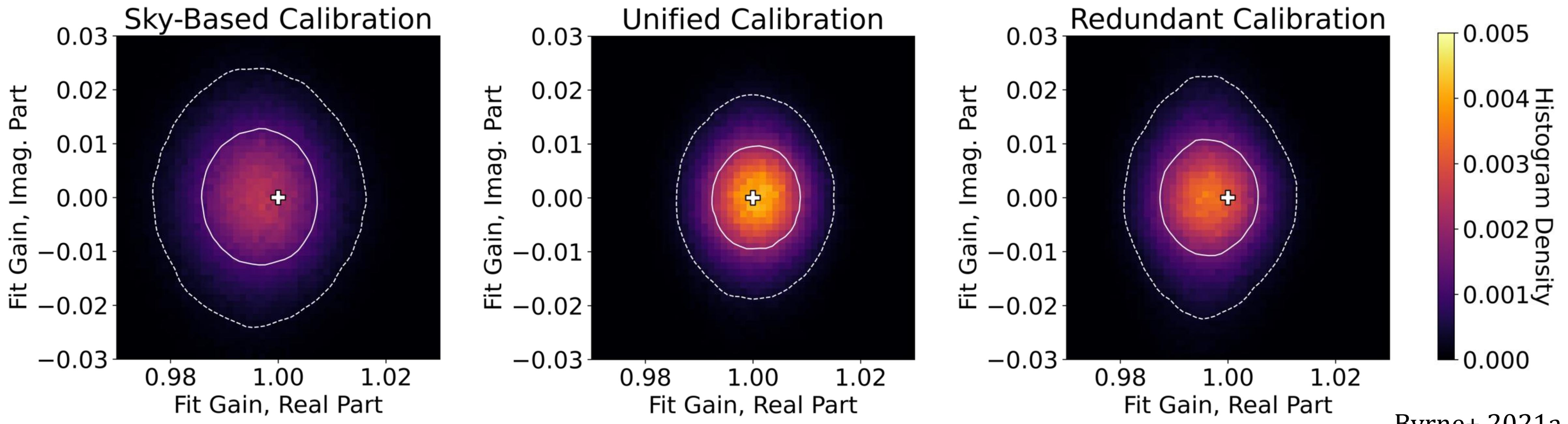
- Operates midway between sky-based and redundant calibration
- Constrains redundant calibration's degeneracies in one step

Unified Calibration Simulation

Calibrated Gains

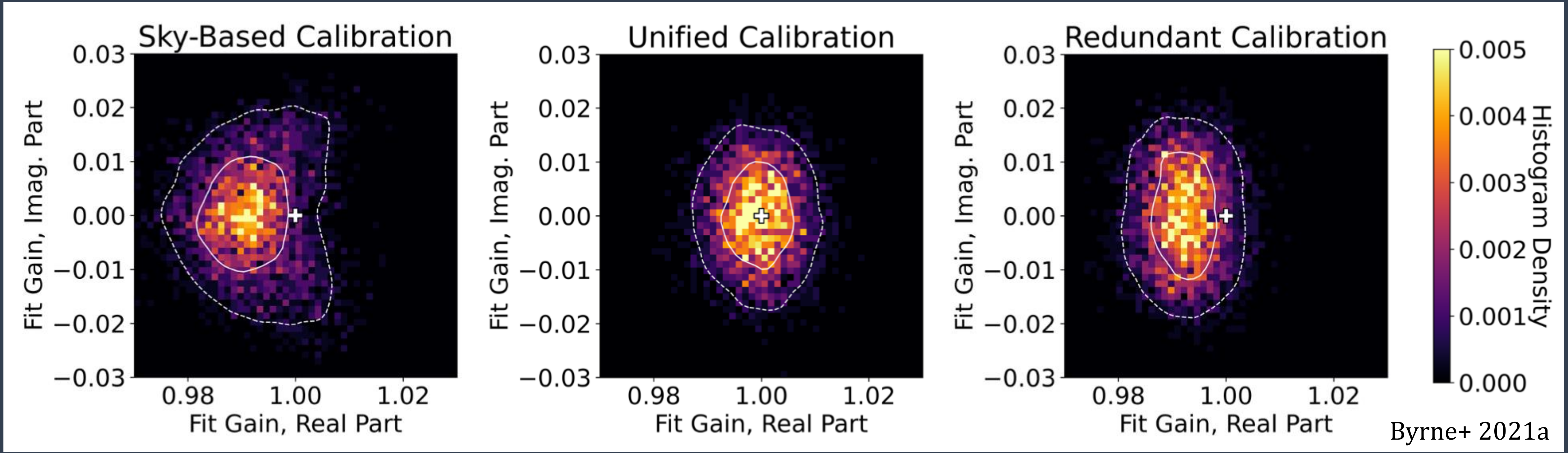


Byrne+ 2021a

- 10,000 calibration trials: 100 realizations each of Gaussian random model visibility error and thermal noise
- Deviations from 1 correspond to calibration errors

Unified Calibration Simulation:
Incomplete Sky Model

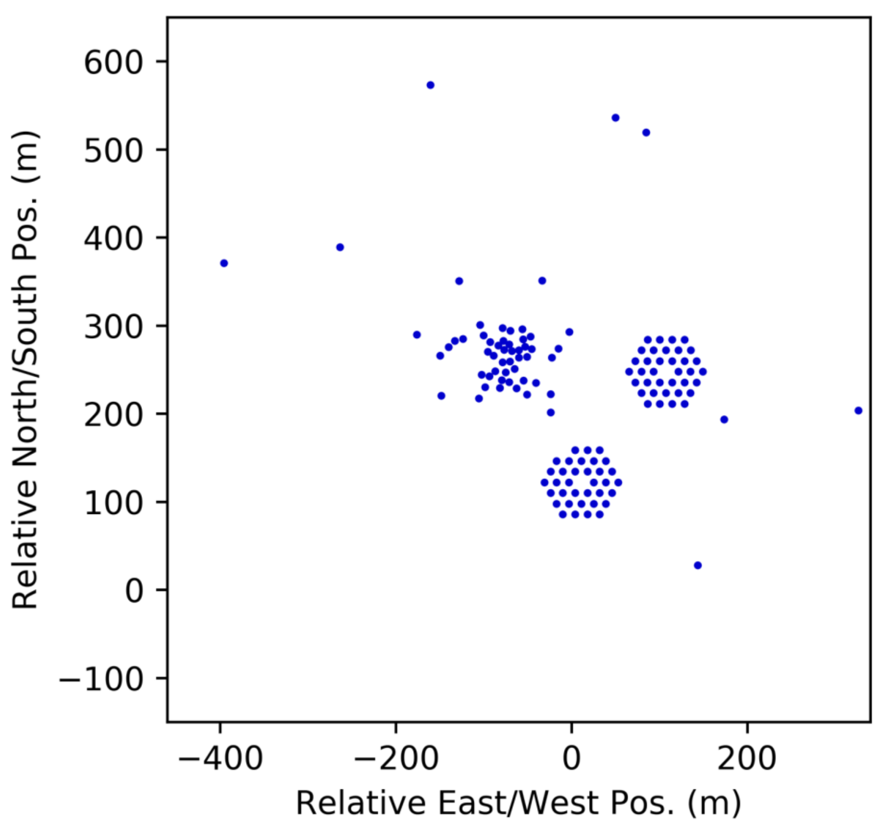
Calibrated Gains



- 100 realizations of thermal noise
- Deviations from 1 correspond to calibration errors

Unified calibration also captures…

- Hybrid arrays with redundant and pseudo-random sub-arrays



MWA Phase II antenna layout

Unified calibration also captures…

- Hybrid arrays with redundant and pseudo-random sub-arrays
- Redundant calibration with imperfect redundancy

HERA

Unified calibration also captures…

- Hybrid arrays with redundant and pseudo-random sub-arrays
- Redundant calibration with imperfect redundancy
- Covariant calibration of baselines that sample correlated modes but are not redundant

MWA compact core

Key Takeaways: Unified Calibration (Byrne+ 2021)

1. Unified calibration performs better than either typical sky-based calibration or redundant calibration
2. Intermediate regime between sky-based calibration and redundant calibration is more accurate and physically-motivated
3. Degeneracies can and should be avoided in calibration optimization
4. We have great flexibility in how we define our calibration framework

Calibration Frameworks

1. Sky-Based Calibration

2. Redundant Calibration

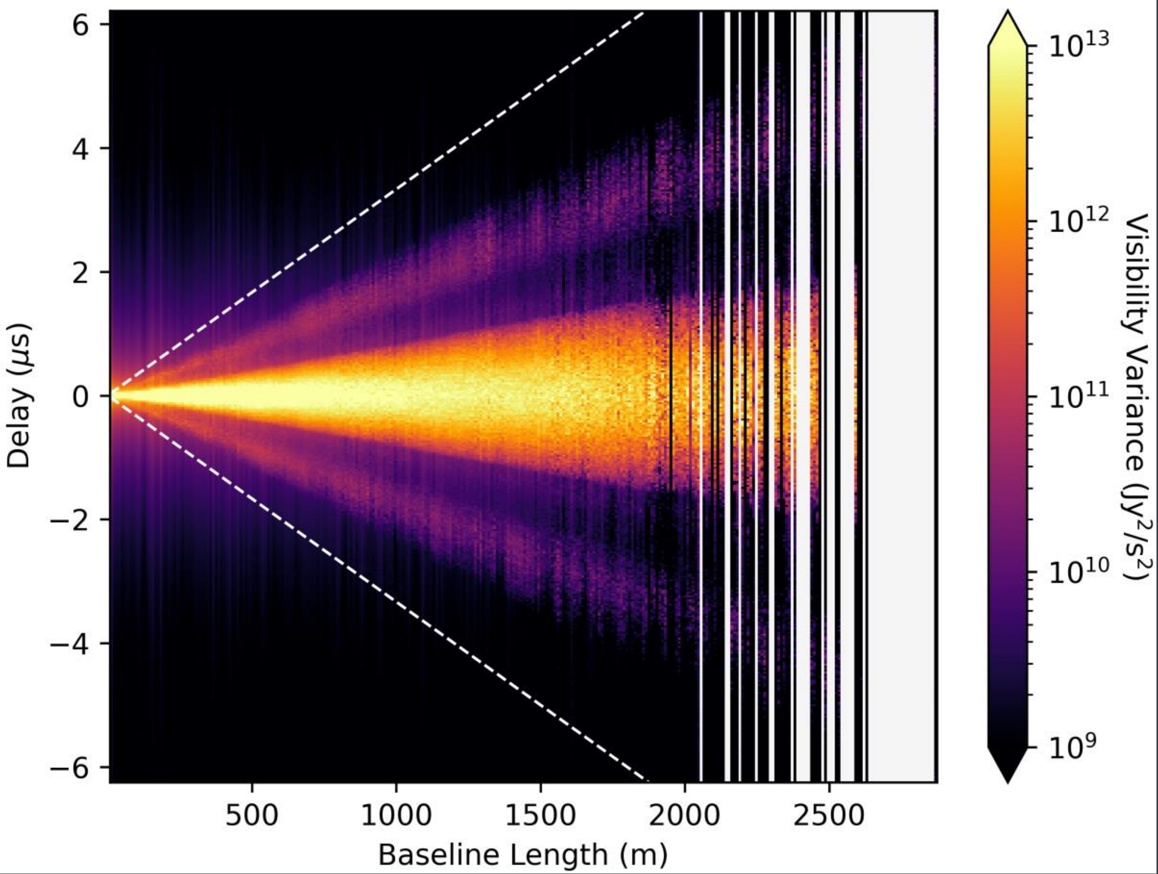
3. Unified Calibration

4. **Delay-Weighted Calibration**

Delay-Weighted Calibration (DWCal) (Byrne 2023)

- Relaxes the assumption that visibilities are uncorrelated across frequency
- Captures the fact that foregrounds are spectrally smooth
- Does not require fitting any additional parameters

Simulated Model Visibility Error for the MWA



Byrne 2023

DWCal Implementation: Mathematical Formalism

Sky-based calibration: $\chi^2(\vec{g}) = \sum_f \sum_{ab} |v_{ab}(f) - g_a(f) g_b^*(f) m_{ab}(f)|^2$

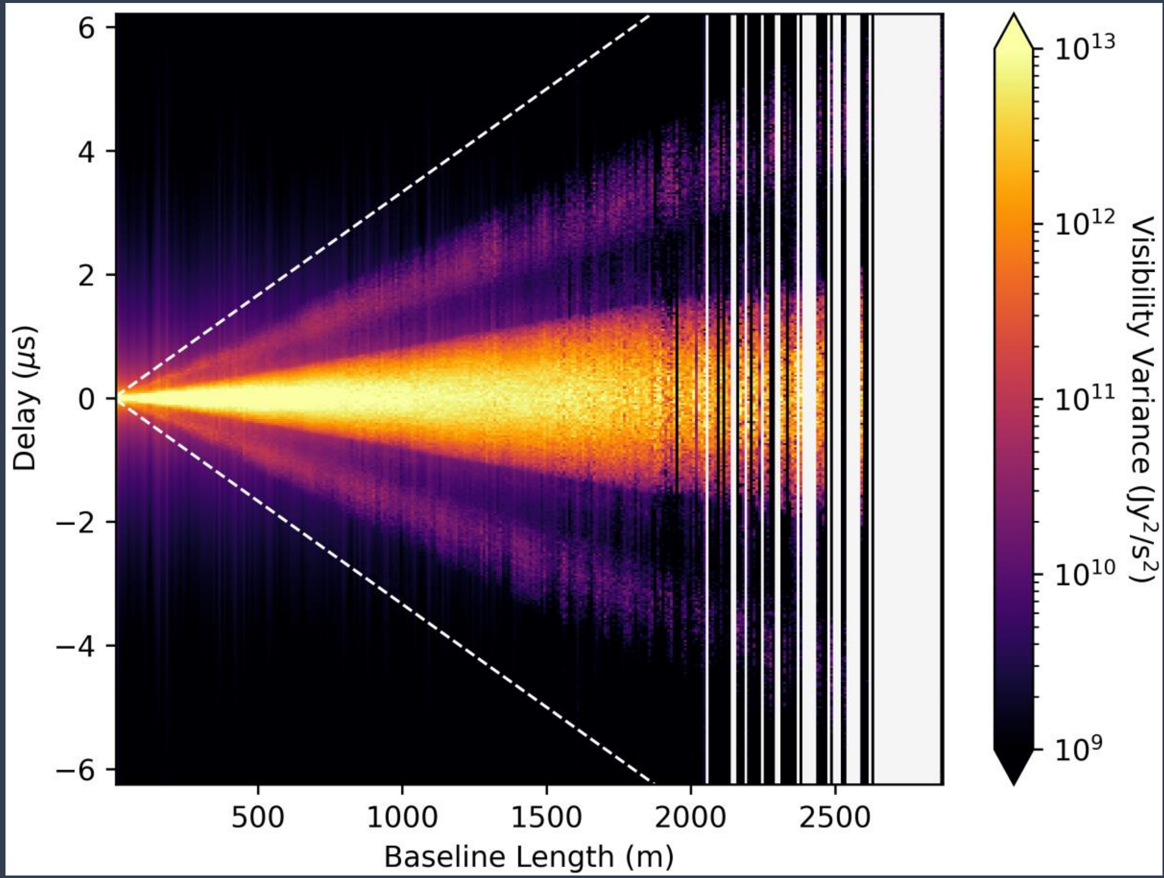
DWCal:

$$\chi^2(\vec{g}) = \sum_f \sum_{f\prime} \sum_{jk} W_{jk}(f - f')[g_j(f) g_k^*(f) v_{jk}(f) - m_{jk}(f)][g_j(f') g_k^*(f') v_{jk}(f') - m_{jk}(f')]^*$$

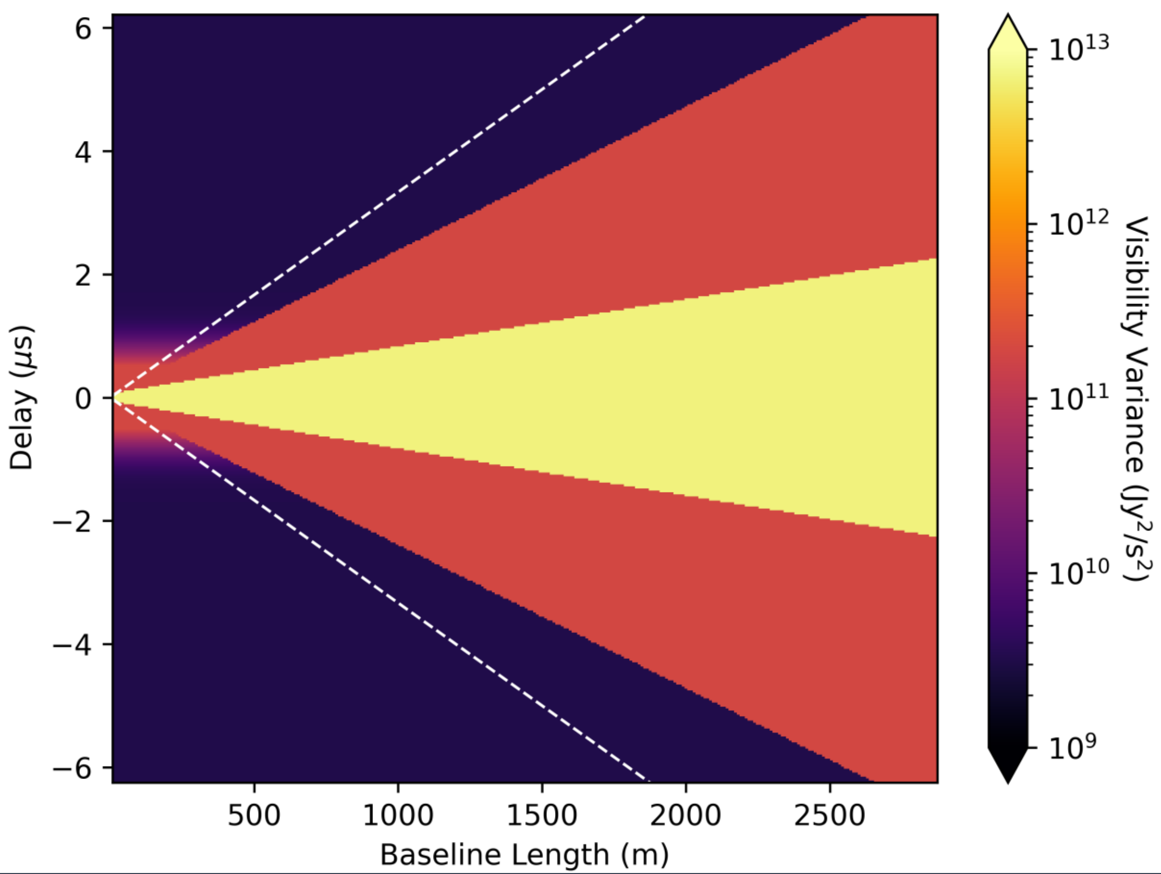
Captures the variance of the model visibilities as a function of baseline and delay

DWCal Implementation: Defining the weighting function

Simulated Model Visibility Error



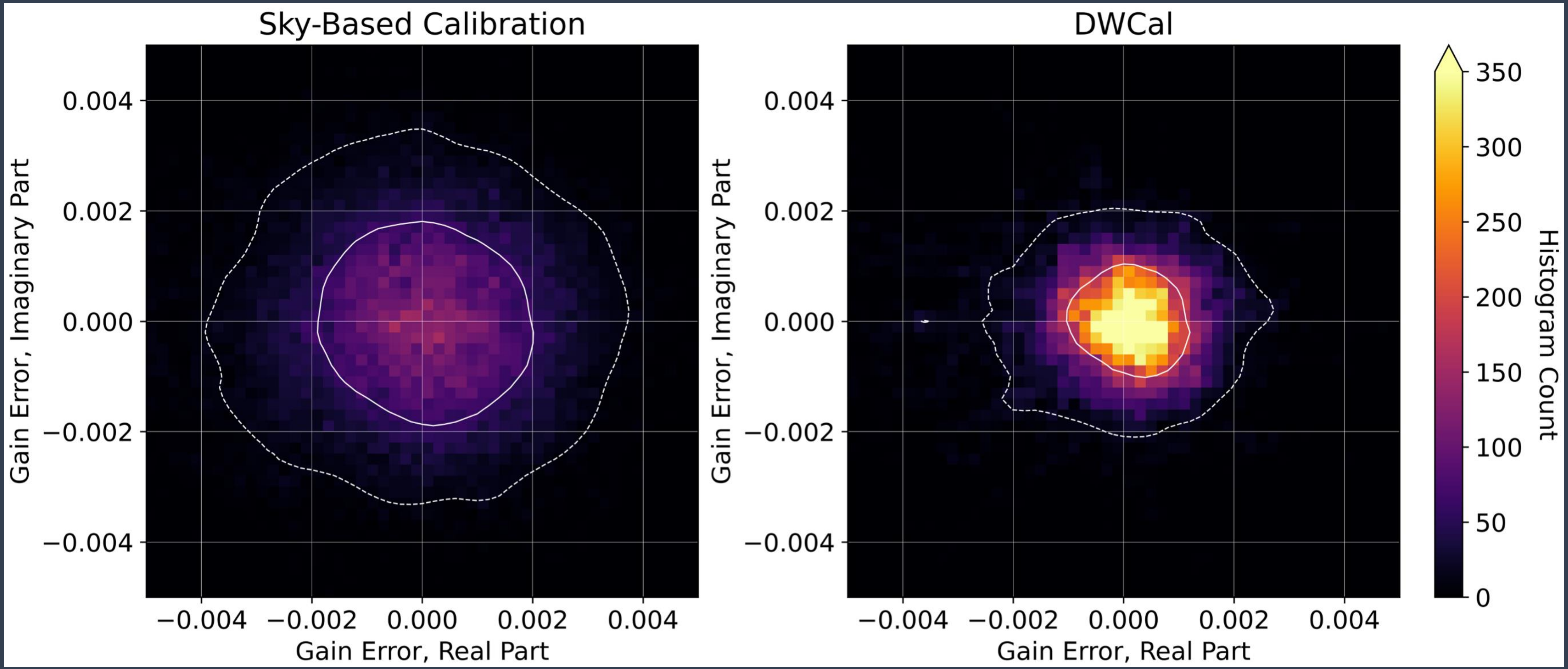
Applied Weighting



Byrne 2023

DWCal Simulation Results

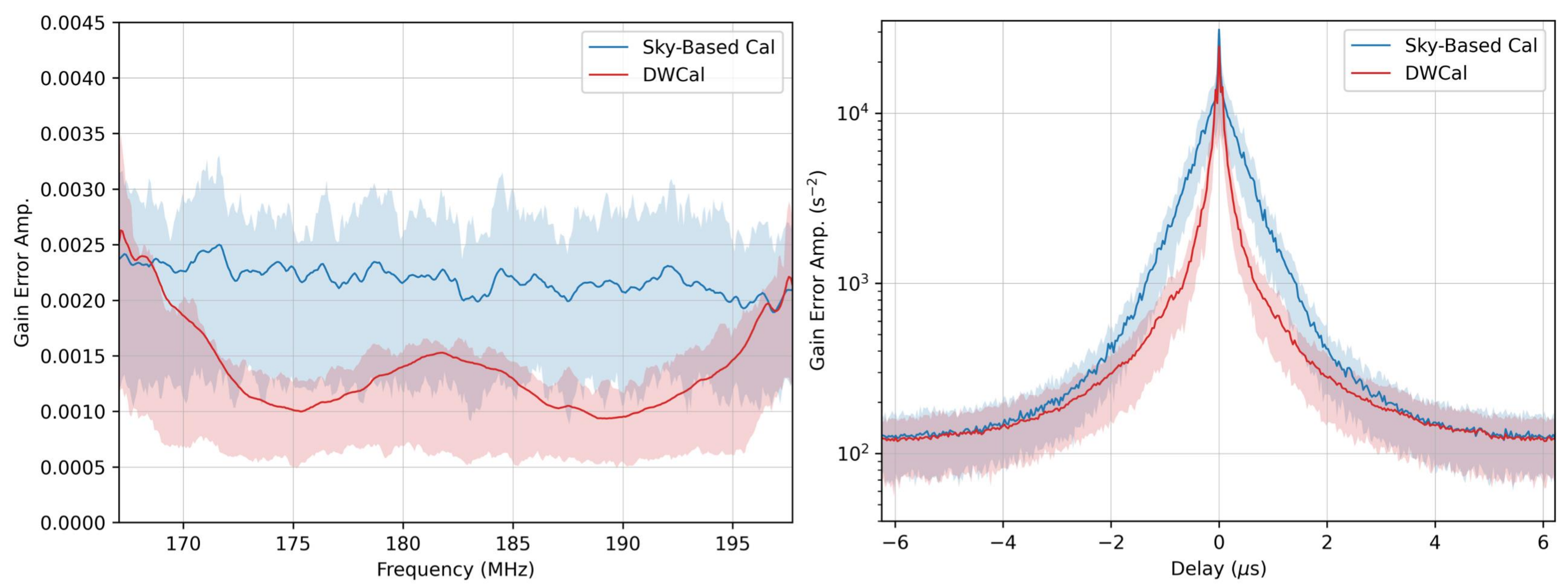
Calibrated Gain Error



Byrne 2023

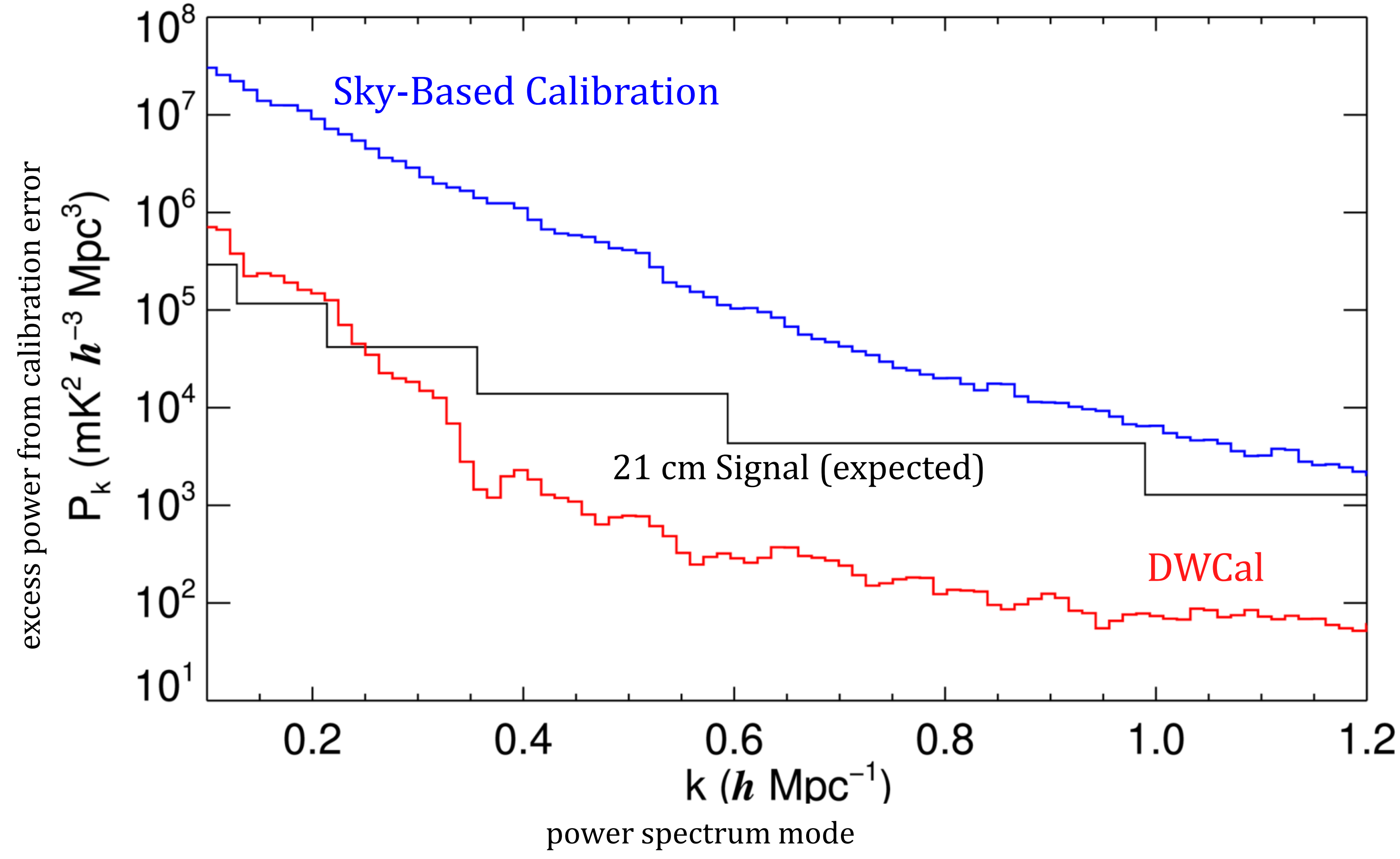
DWCal Simulation Results

Calibrated Gain Error



Byrne 2023

DWCal Simulation Results



Byrne 2023

DWCal can be combined with other calibration approaches

Unified calibration + DWCal

$$\chi^2(\vec{g},\vec{u}) = \sum_f \frac{-1}{2\sigma_\mathrm{T}^2} \sum_j \sum_{\{ab\}\in j} \left|v_{ab}(f) - g_a(f) g_b^*(f) u_j(f)\right|^2 - \sum_f \frac{1}{2\sigma_\mathrm{M}^2} (\vec{u}(f) - \vec{m}(f))^\dagger \mathbf{C}_\mathrm{R} (\vec{u}(f) - \vec{m}(f))$$

$$\chi^2(\vec{g},\vec{u}) = \sum_f \frac{-1}{2\sigma_\mathrm{T}^2} \sum_j \sum_{\{ab\}\in j} \left|v_{ab}(f) - g_a(f) g_b^*(f) u_j(f)\right|^2 - \sum_f \sum_{f'} \sum_{jk} \frac{1}{2\sigma_\mathrm{M}^2} W_{jk}(f - f') \left[(\vec{u}(f) - \vec{m}(f))^\dagger \mathbf{C}_\mathrm{R} (\vec{u}(f) - \vec{m}(f))\right] \left[(\vec{u}(f') - \vec{m}(f'))^\dagger \mathbf{C}_\mathrm{R} (\vec{u}(f') - \vec{m}(f'))\right]^*$$

Key Takeaways: DWCal (Byrne 2023)

1. DWCal performs much better than typical sky-based calibration
2. More accurate calibration does not require fitting more parameters
3. Primary drawback is that DWCal is not embarrassingly parallel in frequency

Next Steps

- Implementing DWCal for the OVRO-LWA ($z \sim 17$)
- Implementing hybrid redundant calibration with DWCal for HERA ($z \sim 7$)
- Designing future experiments and analysis pipelines around calibratability with advanced techniques
- Advances in calibration optimization algorithms
  - Newton's method solving
- Further advances in precision analysis beyond calibration