清華大學天文系
Department of Astronomy, Tsinghua University

Constrain Primordial non-Gaussianity with the 21 cm Power Spectrum & Bispectrum from the Epoch of Reionization

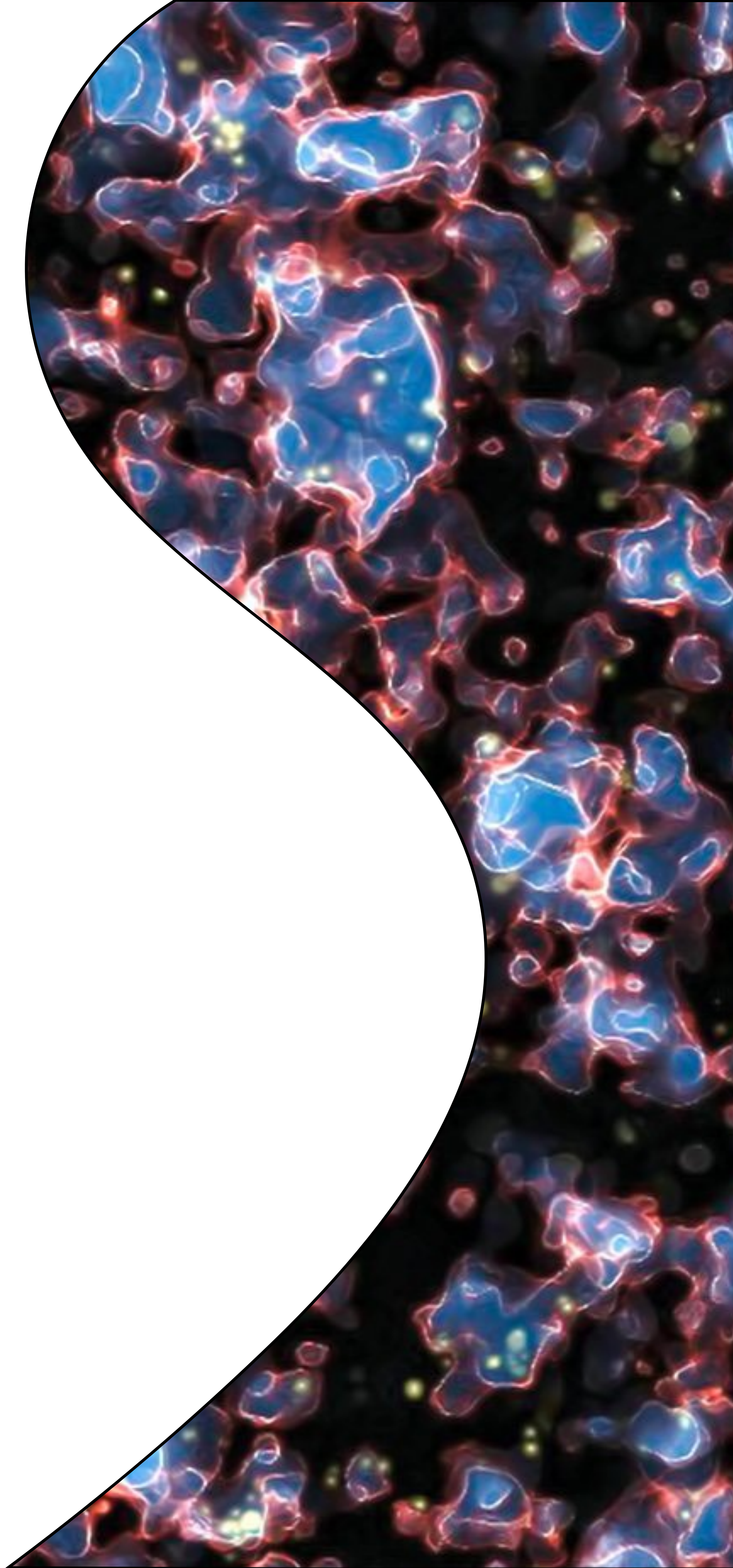
Siyi Zhao (赵思逸, Tsinghua)

Collaborators: Prof. Yi Mao (茅奕, Tsinghua), Zhenyuan Wang (王震远, Penn State)

21 cm Cosmology Workshop 2023 & Tianlai Collaboration Meeting

2023.7.19 Shenyang

Content



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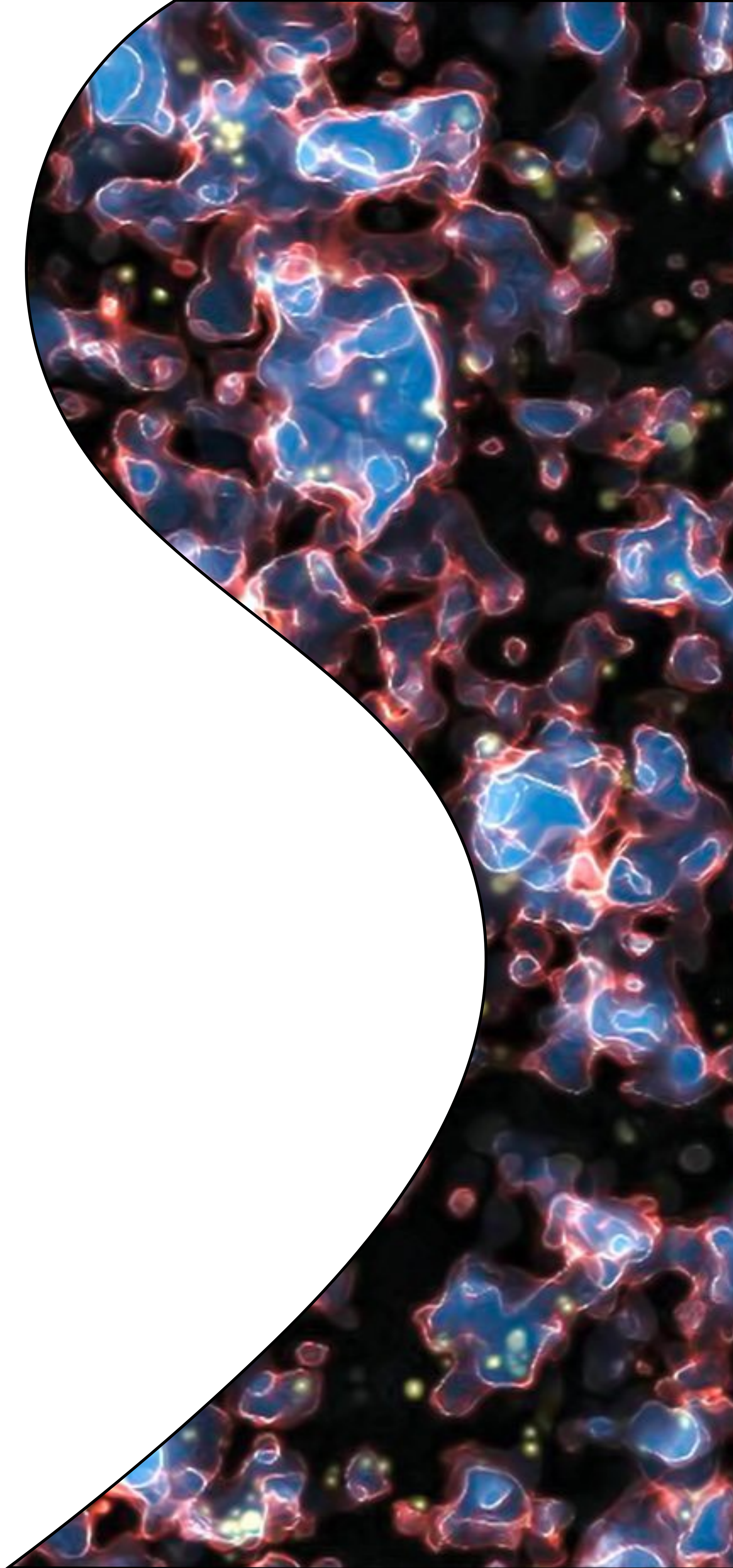

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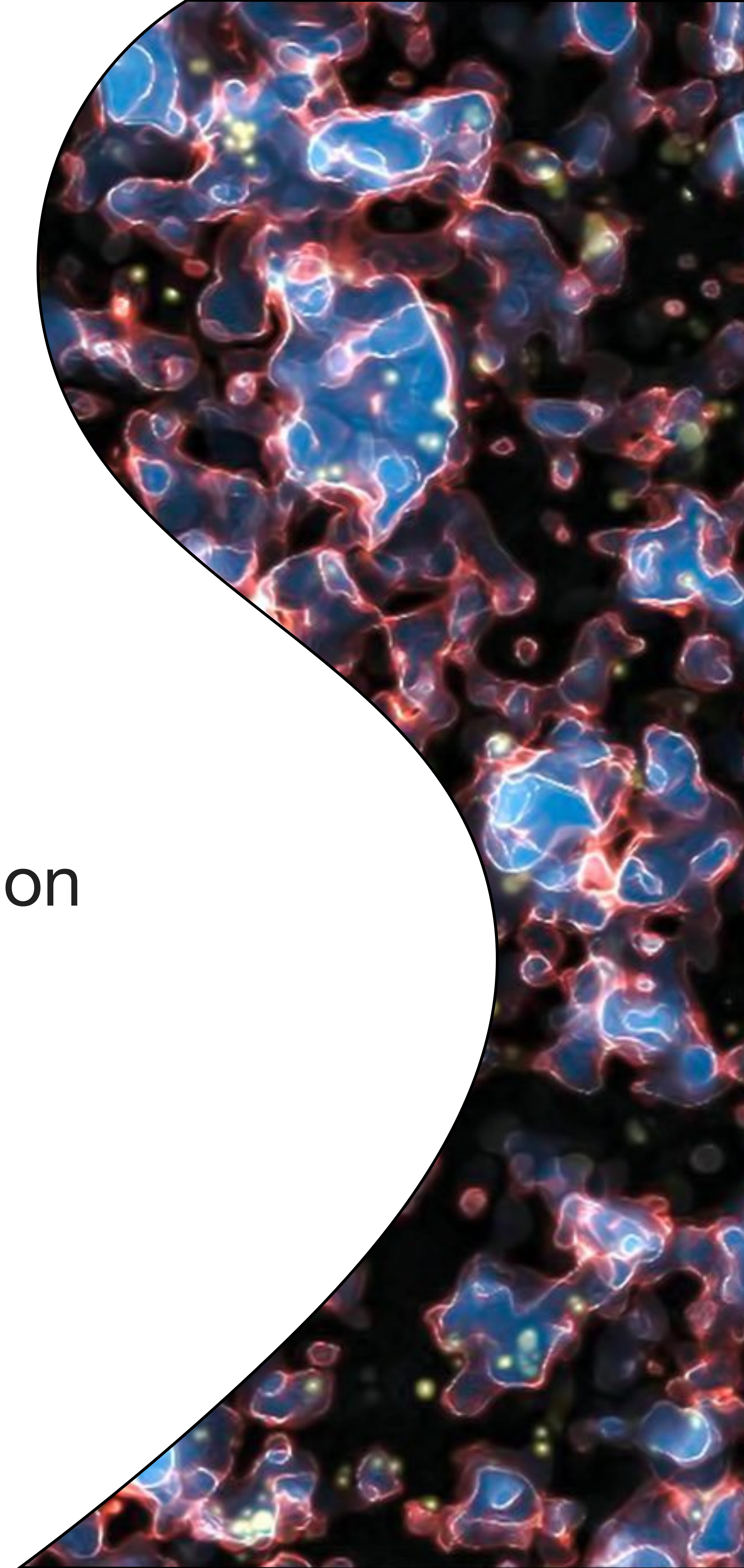
  - PNG Signature in 21-cm Bispectrum



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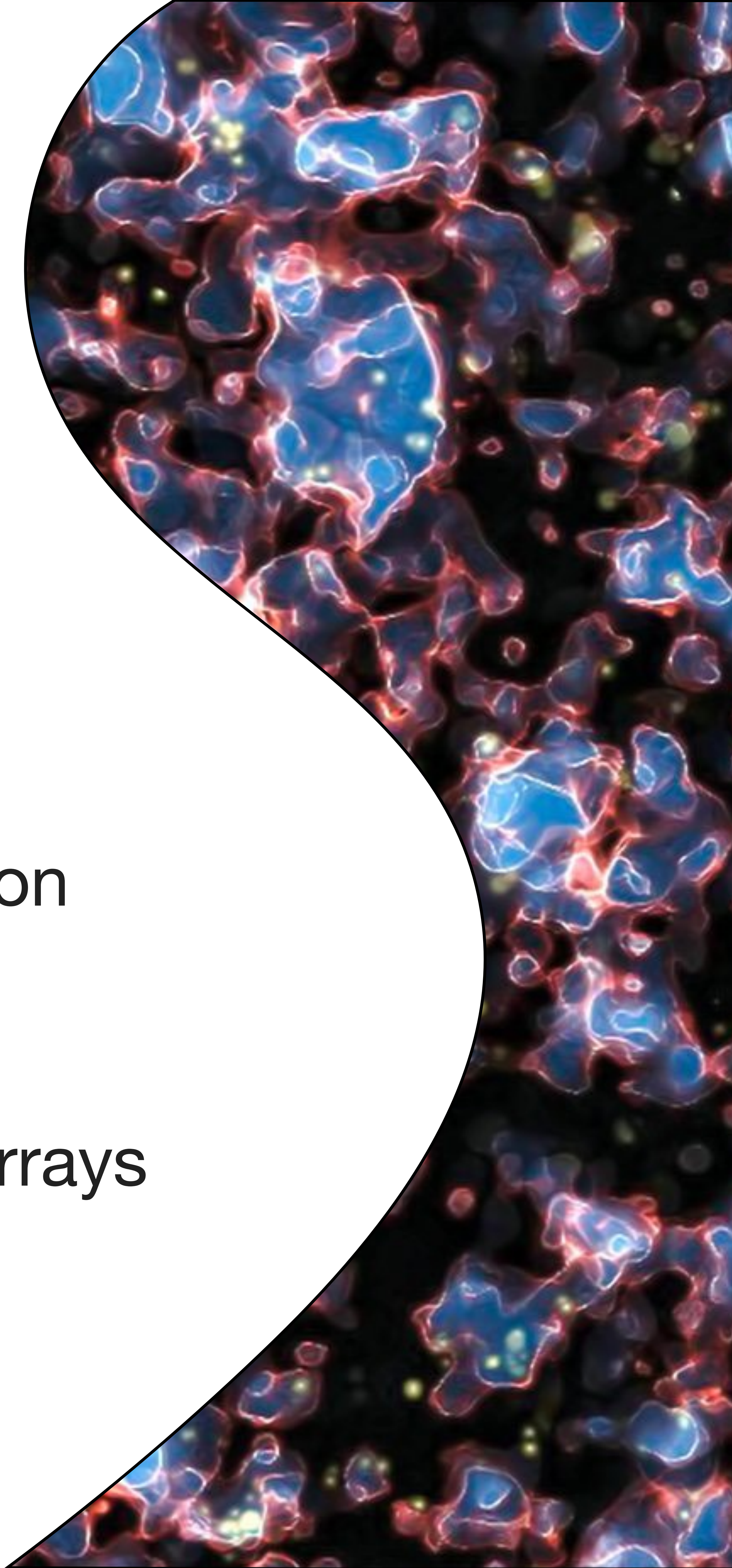

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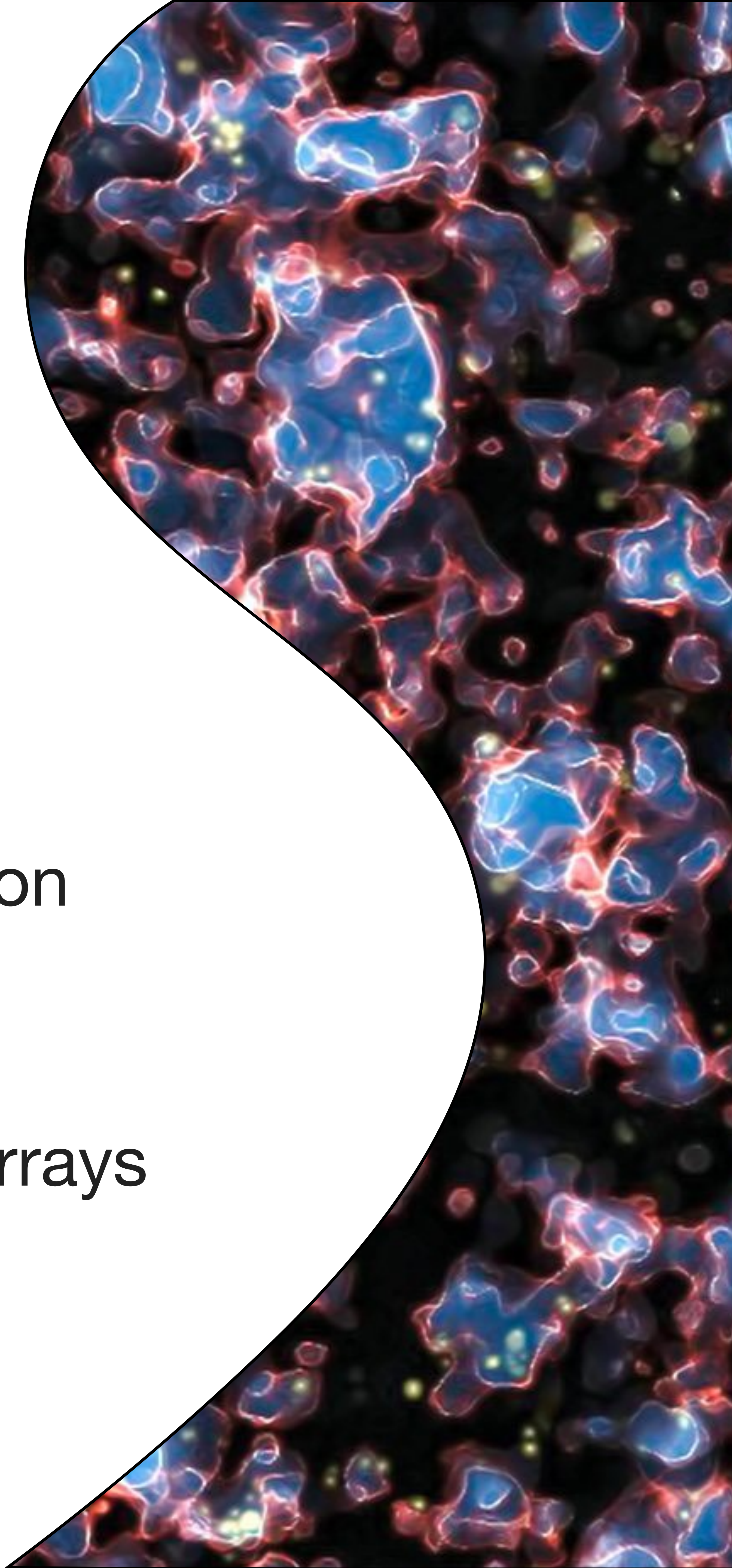
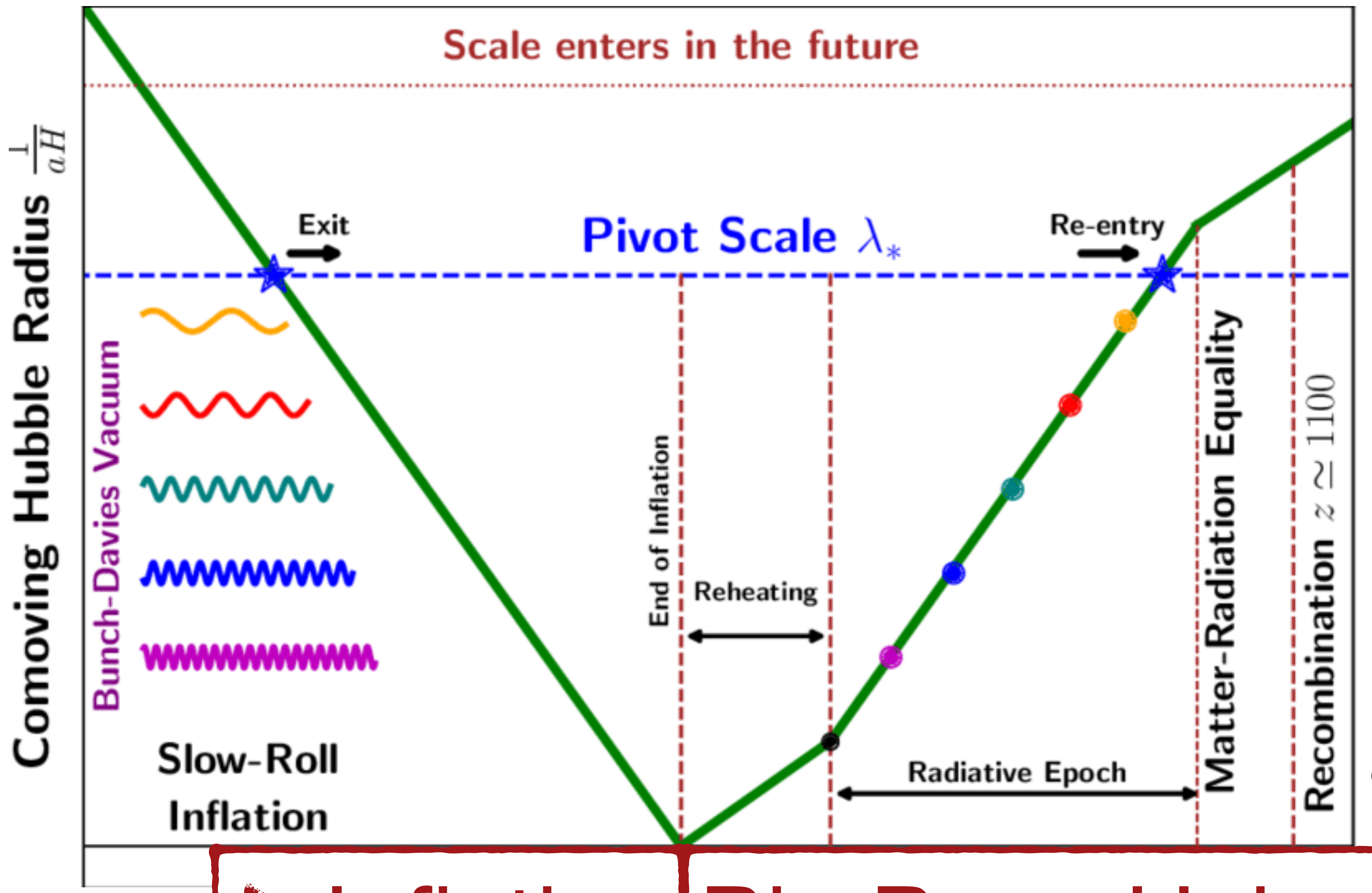
- Summary

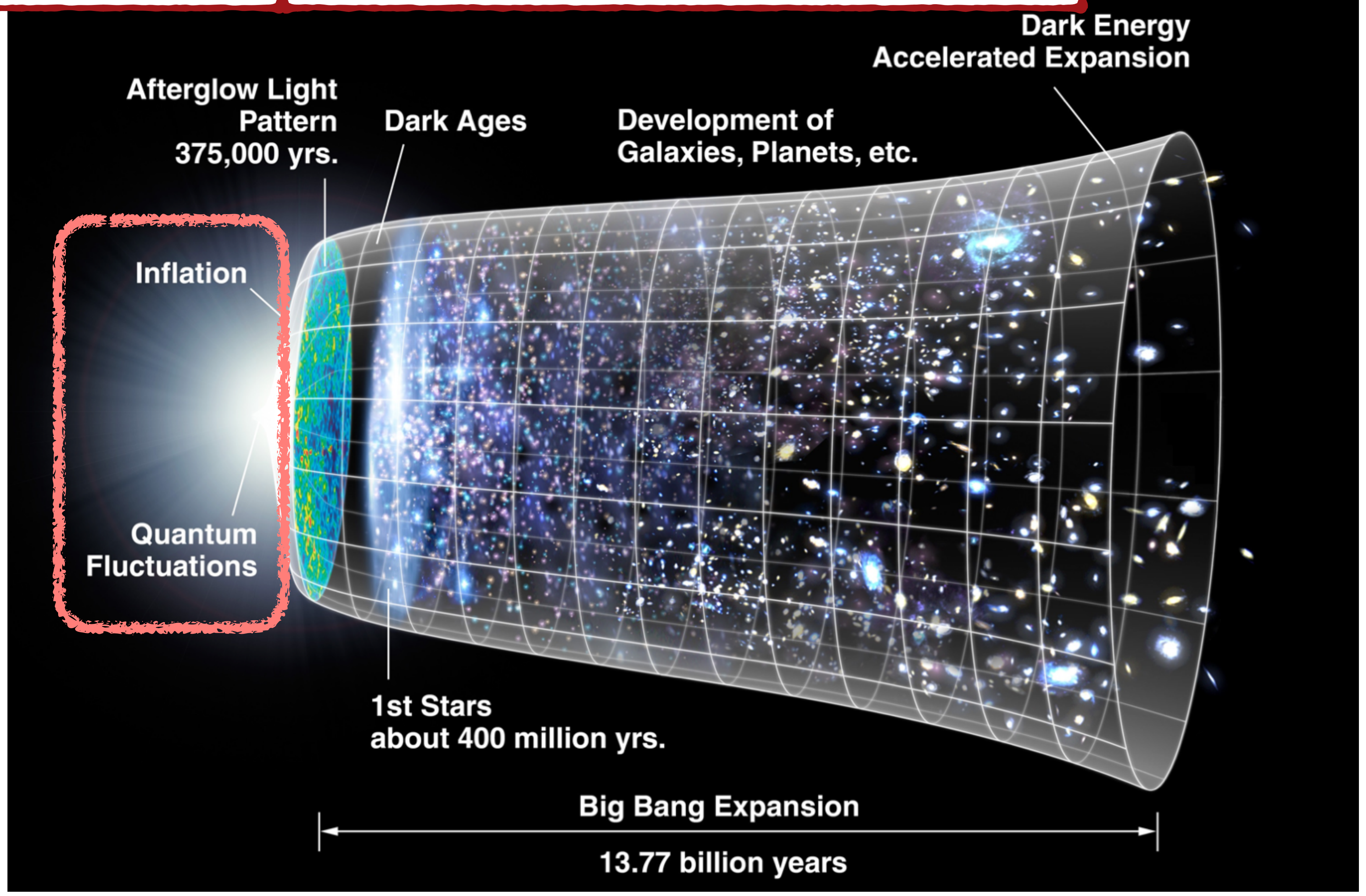


Primordial Fluctuations



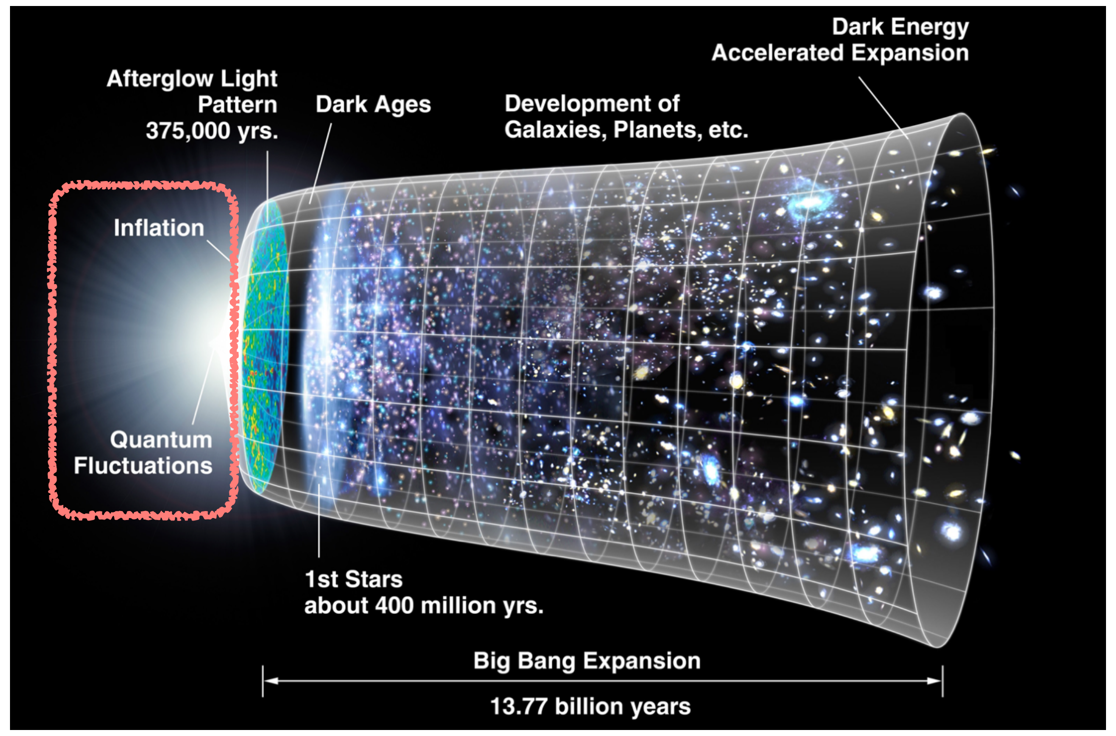
adapted from Mishra & Sahni, 2022

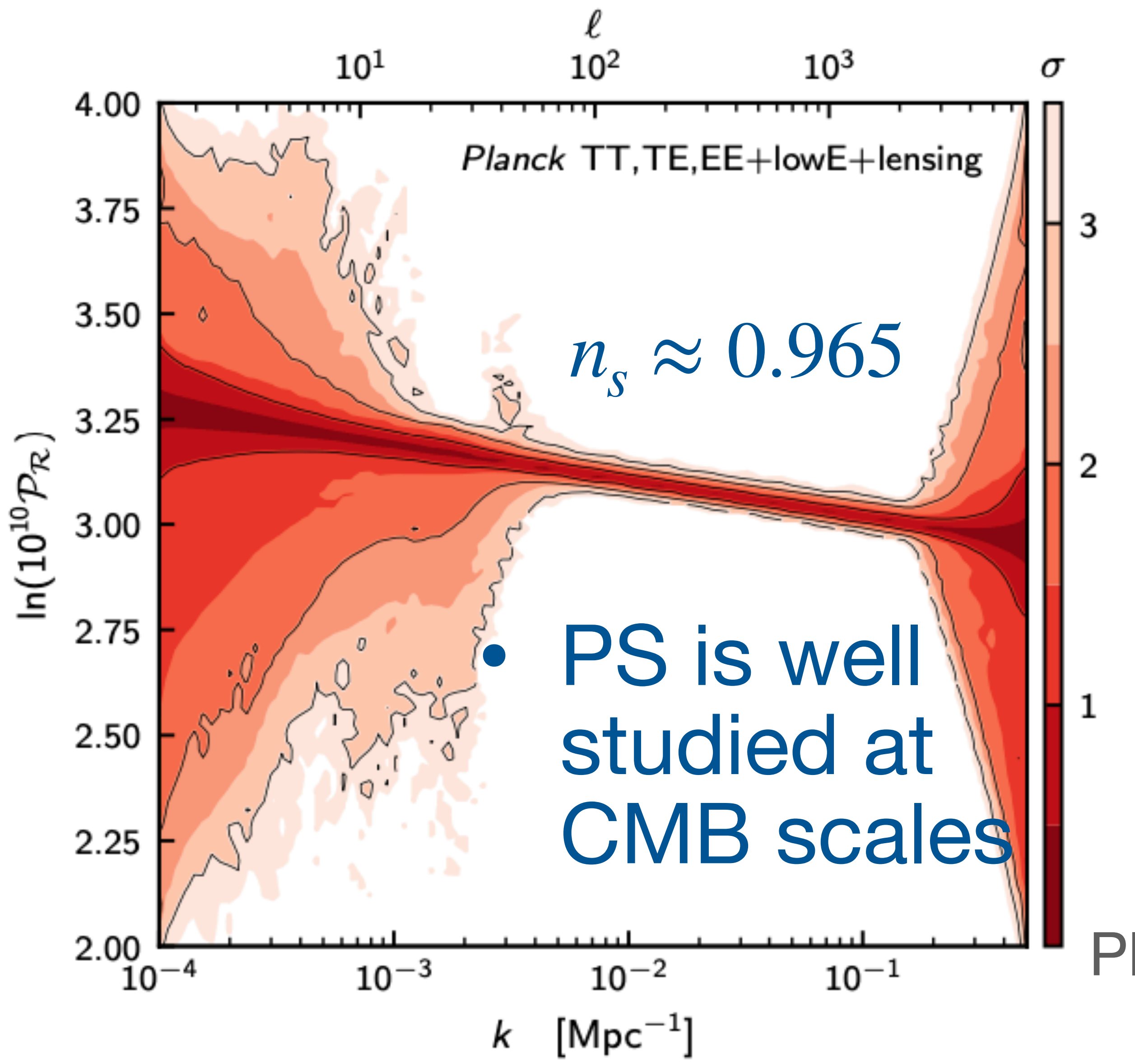
Quantum Fluctuations → Inflation | Big Bang Universe → Seed of large scale structures(LSS)



Primordial non-Gaussianity and Bispectrum(BS)

Inflation models predict different values of fNL.





Planck 2018

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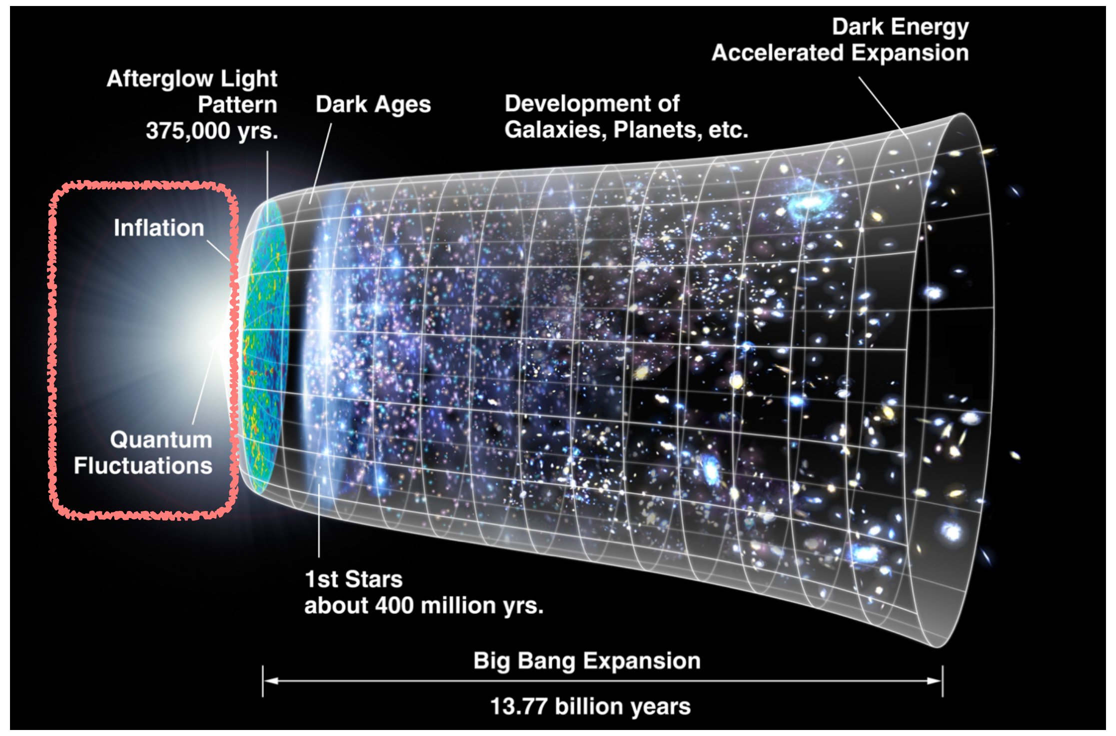

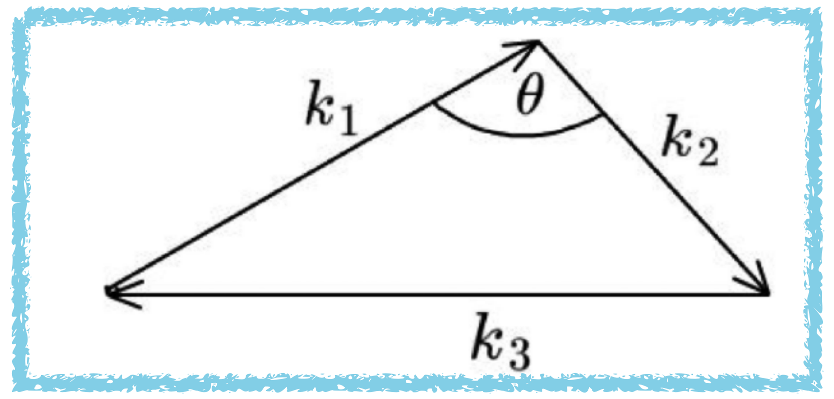

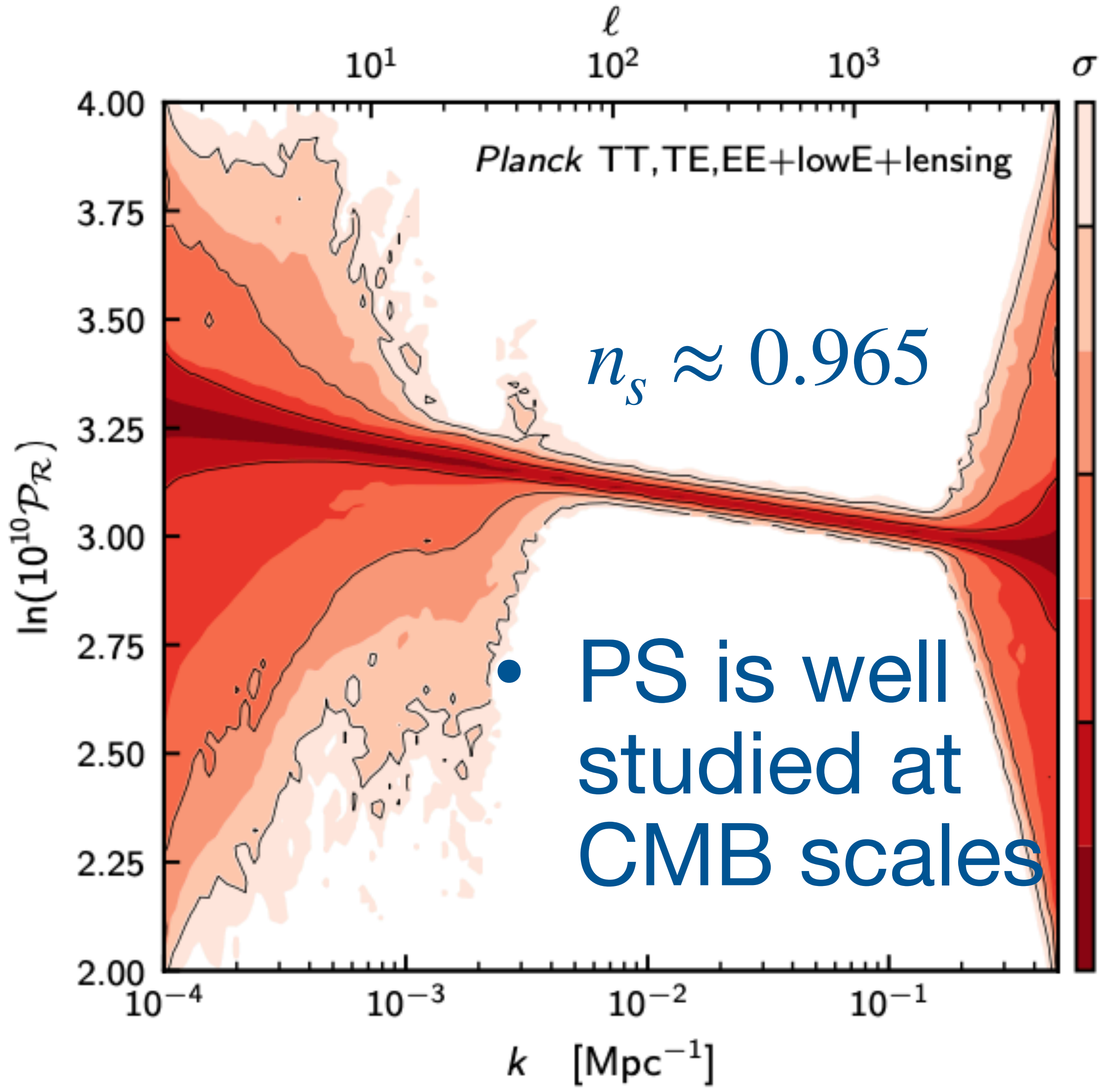

Planck 2018

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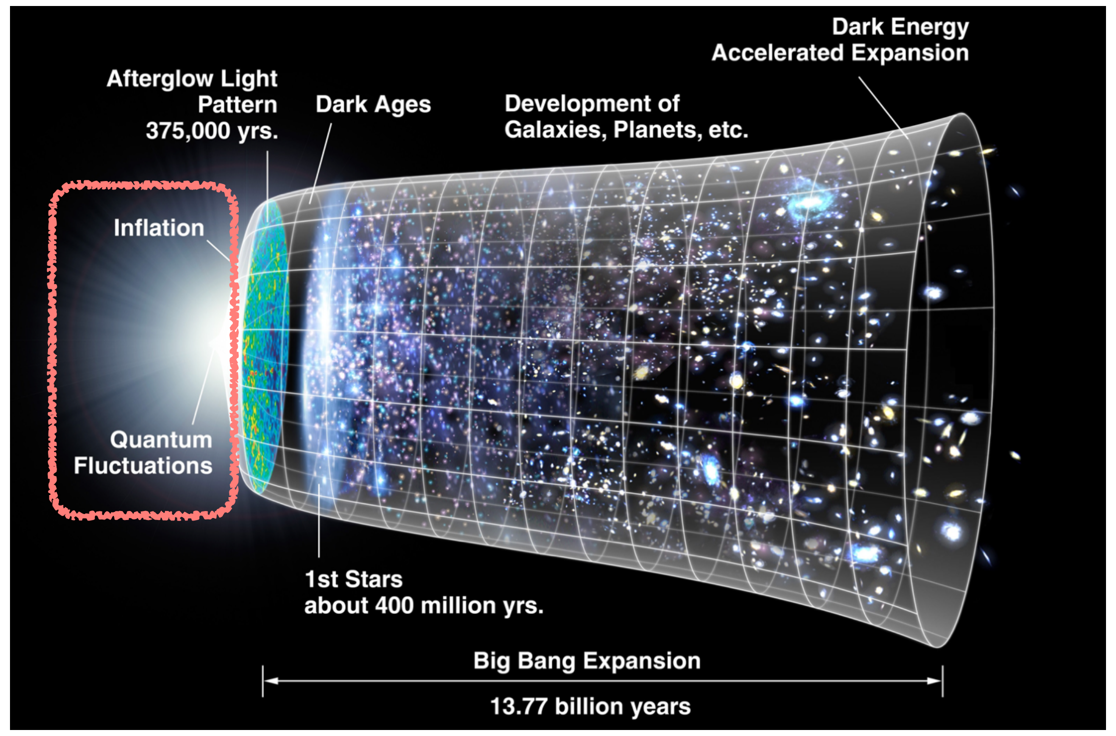
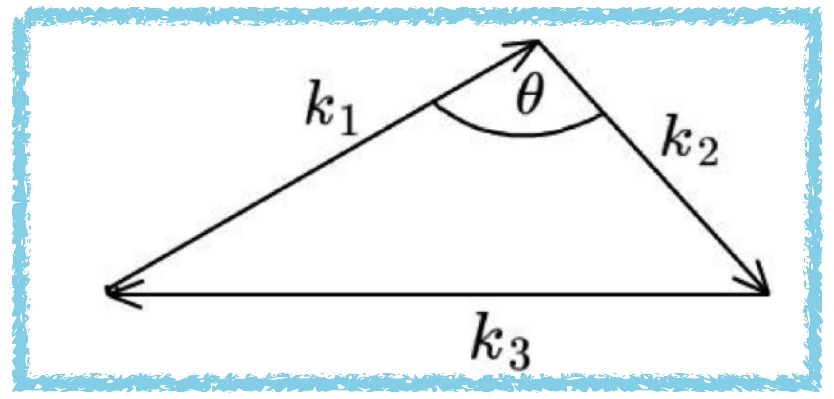
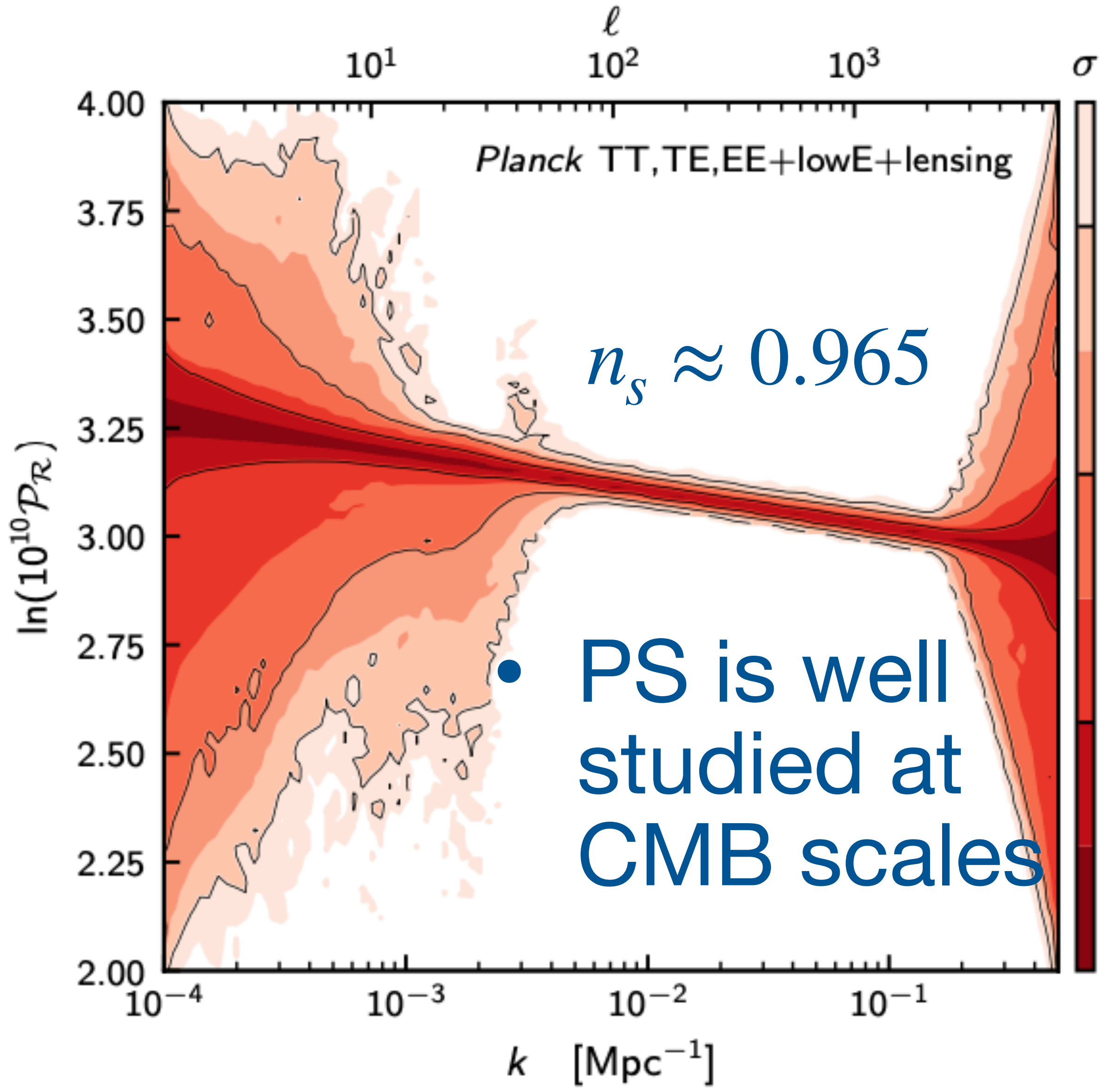




Primordial Non-Gaussianity

- in the local template: $\phi(x) = \phi_G(x) + f_{\rm NL}\left[\phi_G^2(x) - \langle \phi_G^2(x) \rangle\right]$



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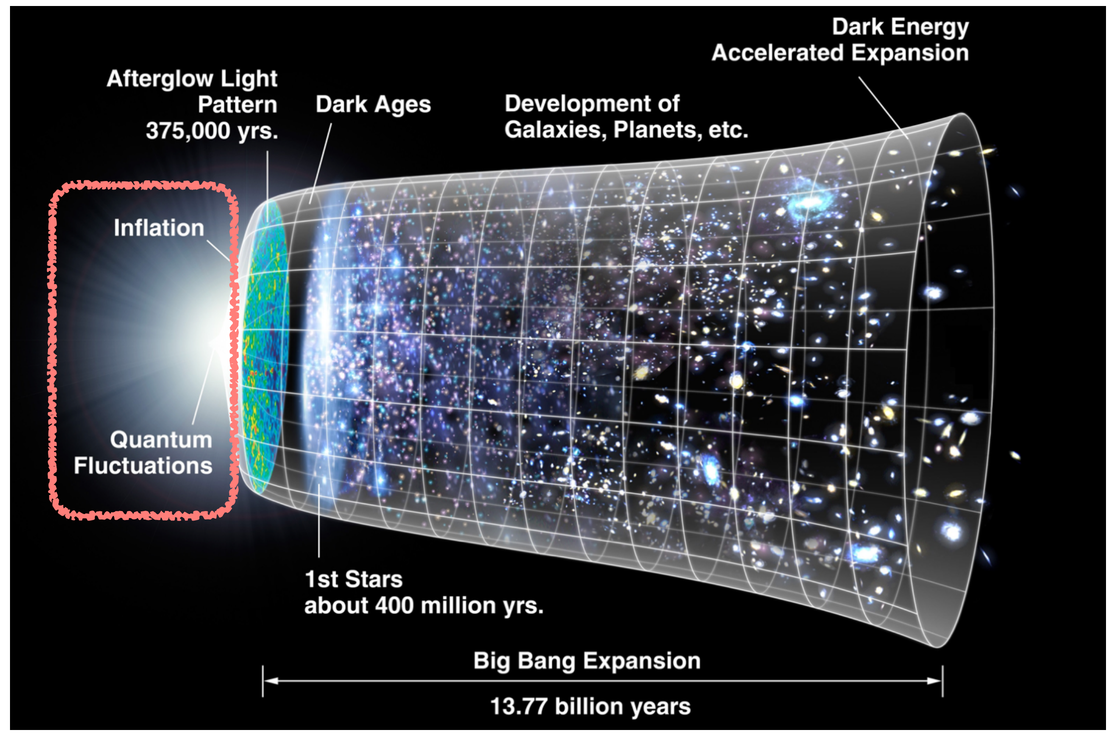

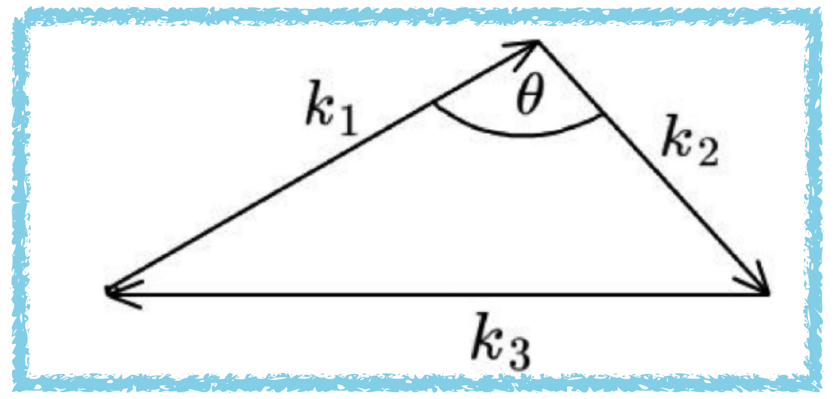

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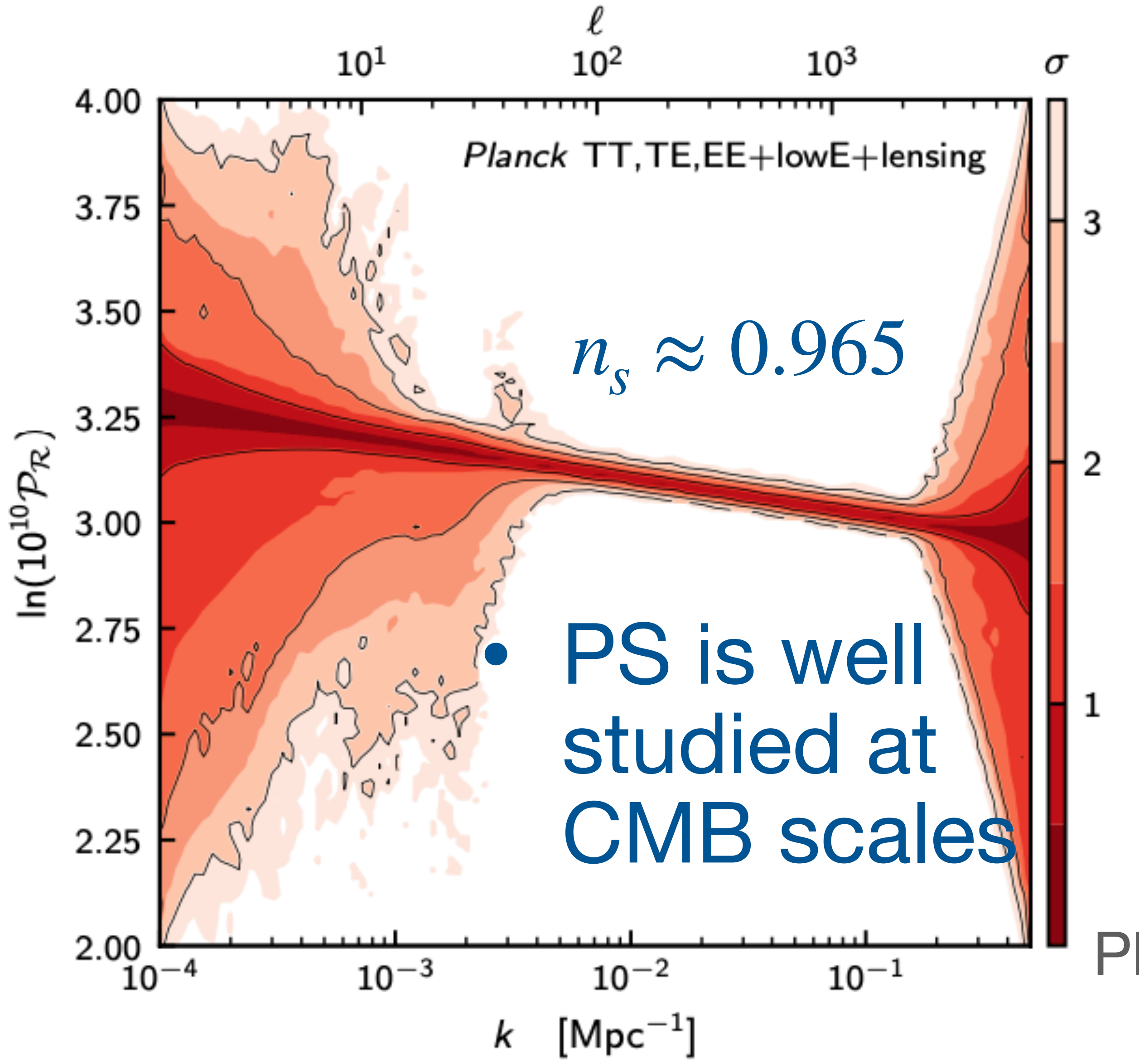

Planck 2018

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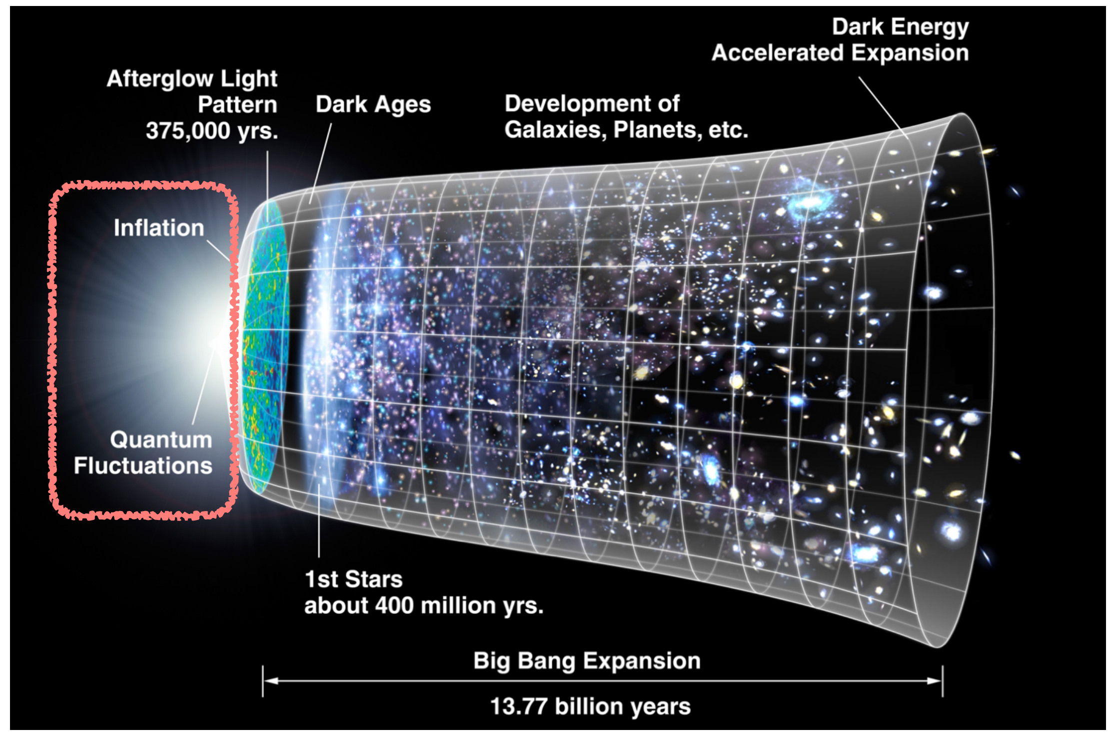

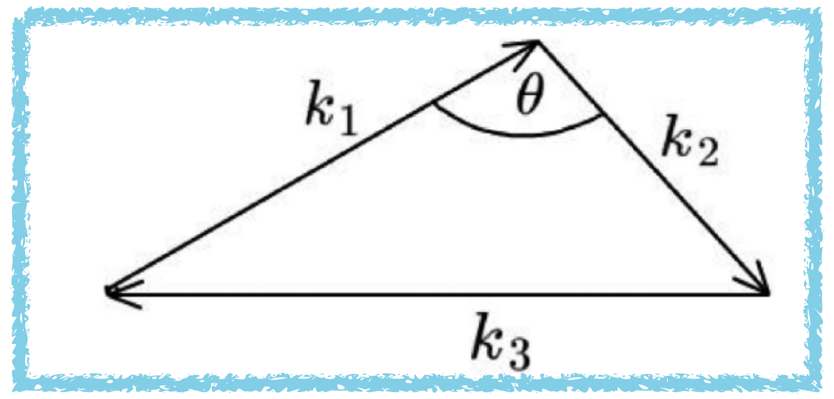

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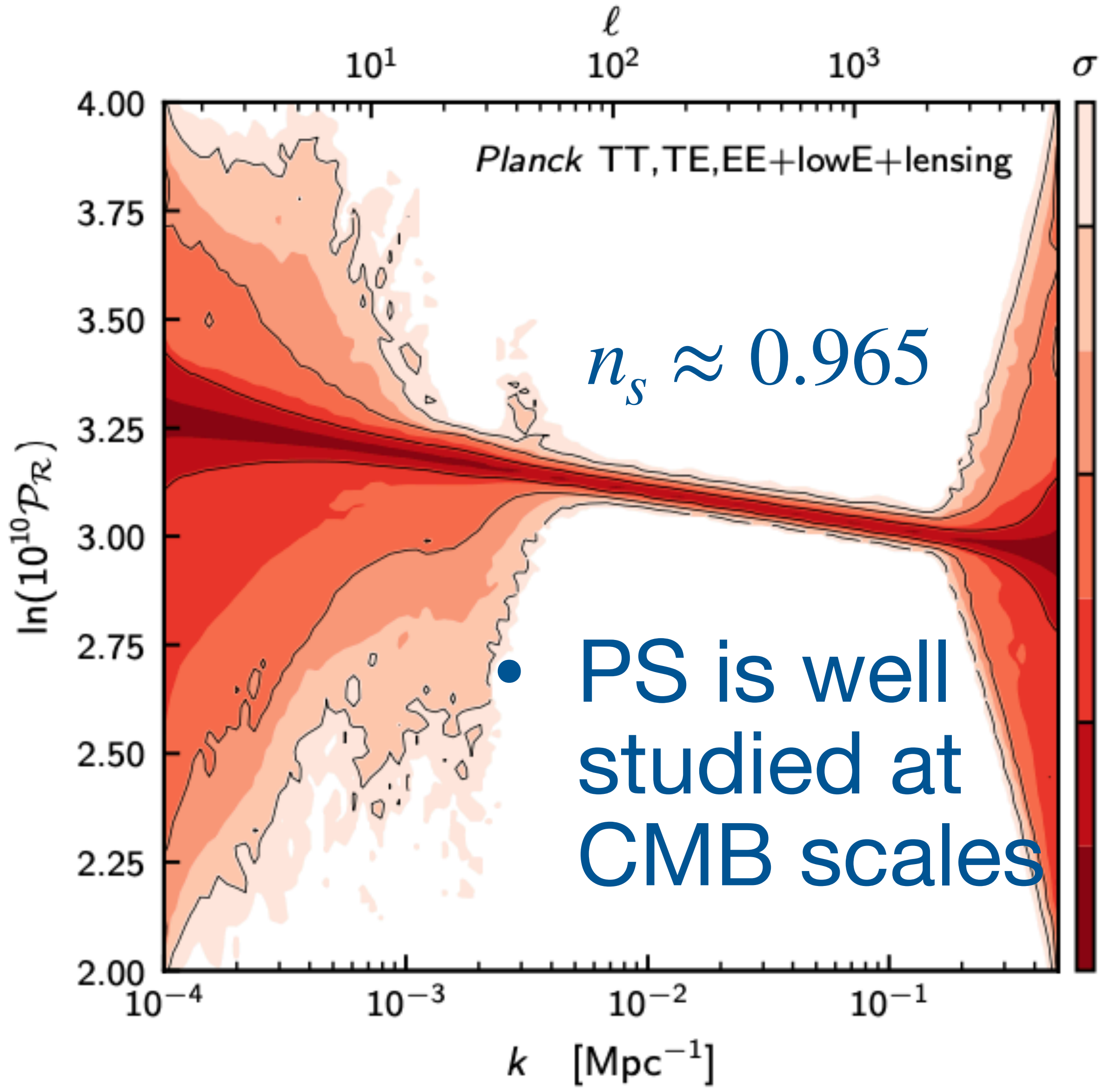
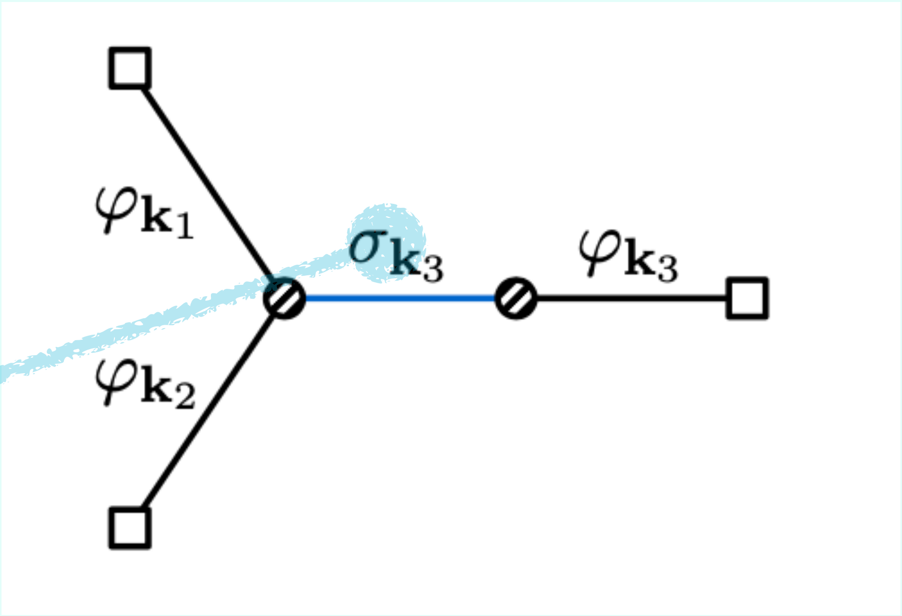


- PS is well studied at CMB scales

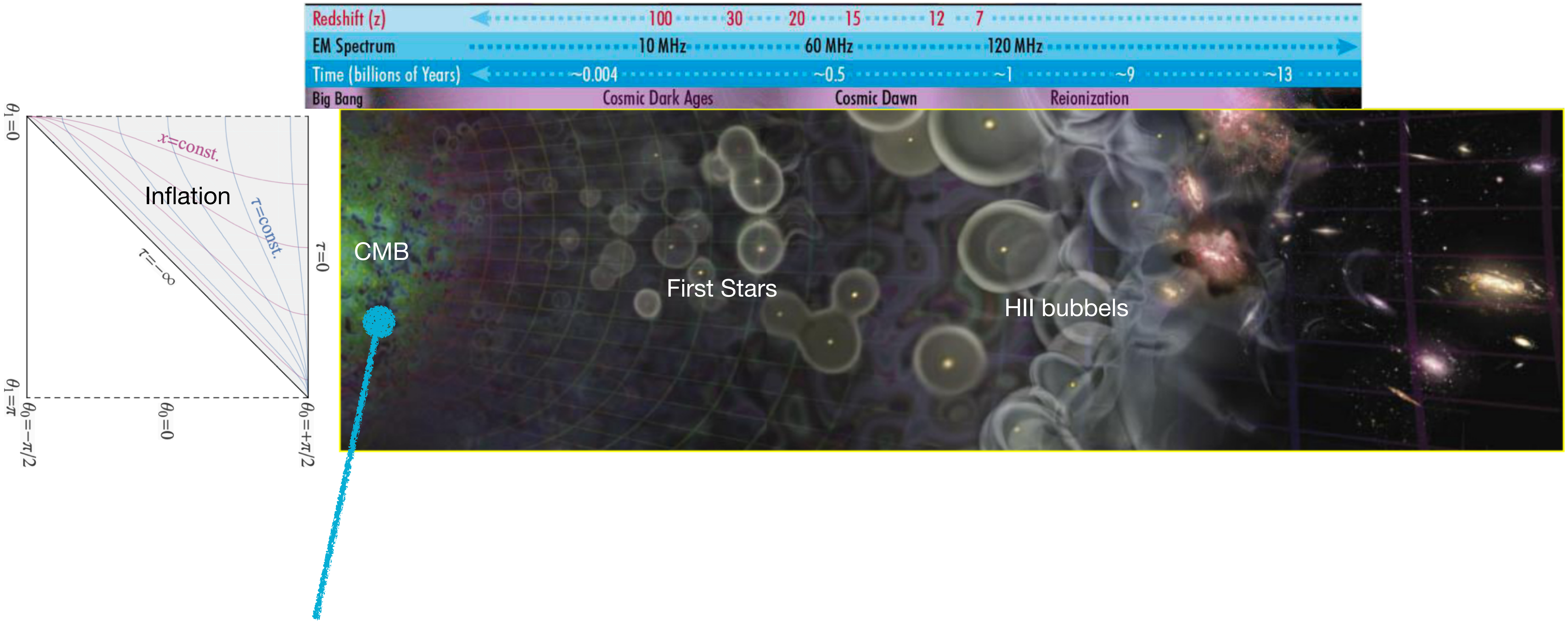
Planck 2018

- Single field, gravity of inflaton
  - consistency relation $f_{\rm NL} = \frac{5}{12}(1 - n_s) \approx 0.015$
- More complicate physics:
  - other self-interaction of inflaton
  - couplings to other fields



Detect PNG with CMB

CMB is the best tracer up to now.



adapted from A. Loeb, 2006, *Scientific American*, 295, 46

CMB bispectrum measurement $f_{\rm NL} = 0.9 \pm 5.1$ (Planck18)

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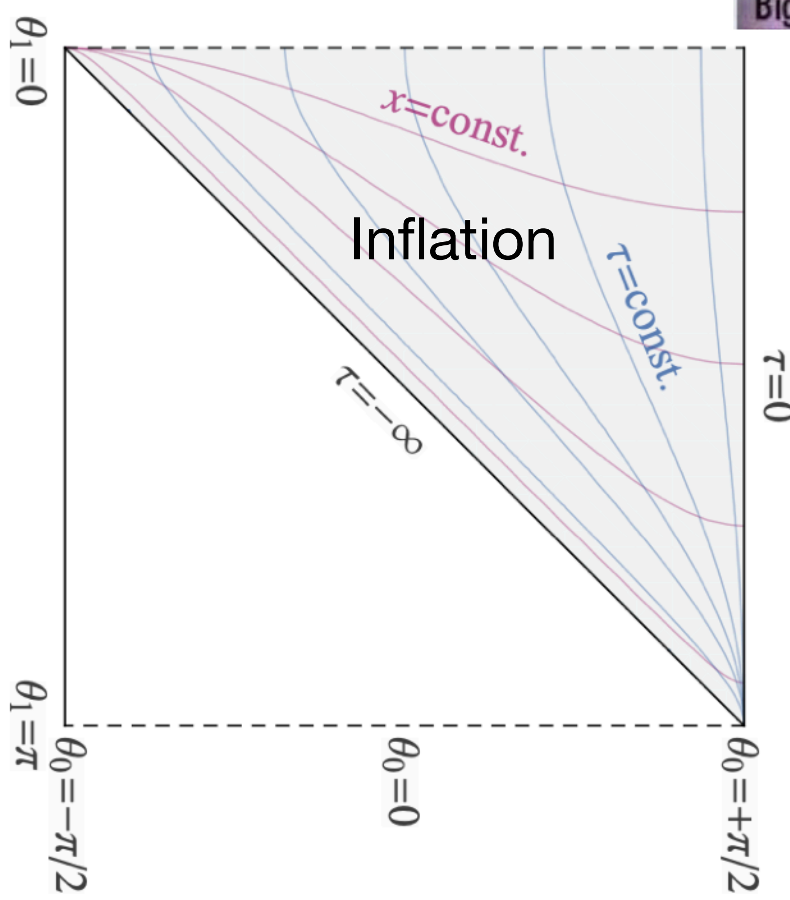

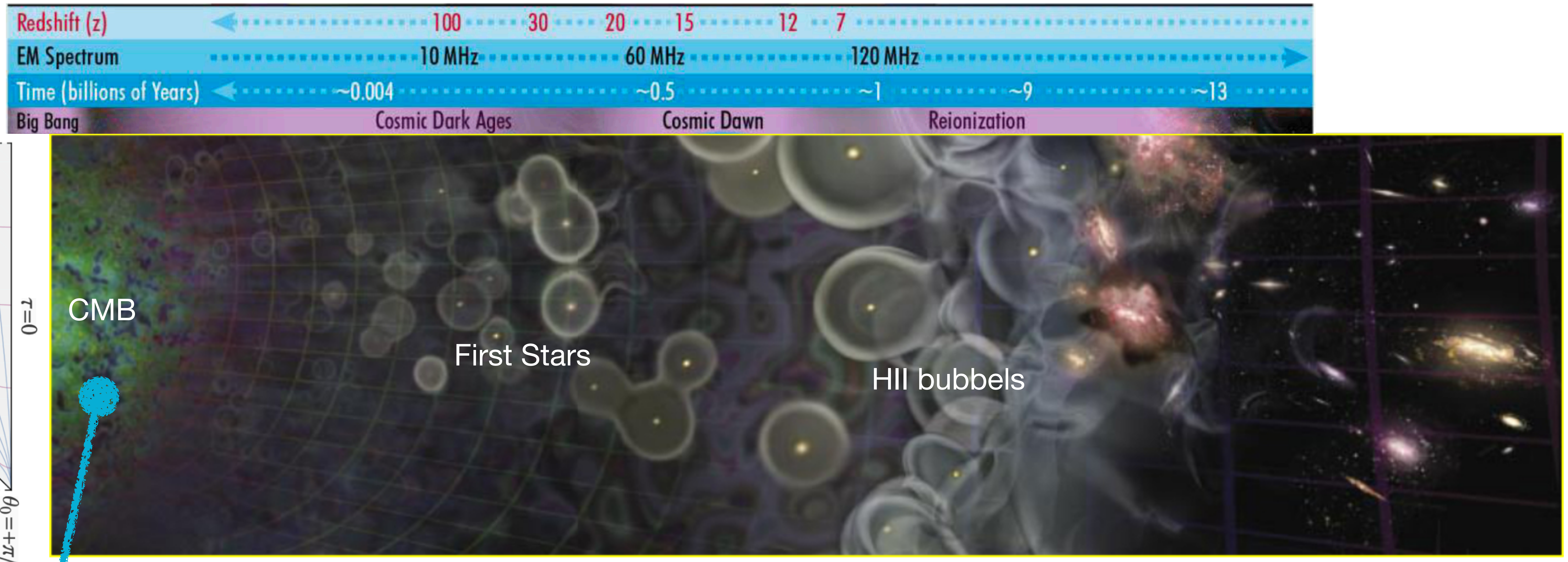

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cosmic variance limit: $\Delta f_{\rm NL} \gtrsim 1.6$ (Komatsu & Spergel, 2001)

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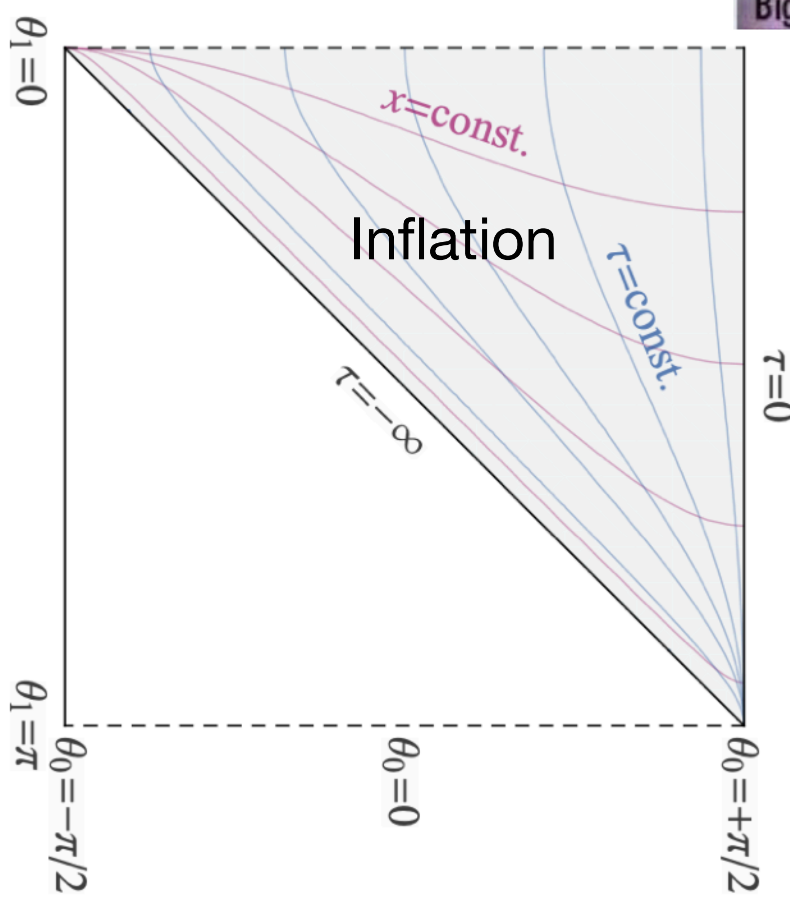

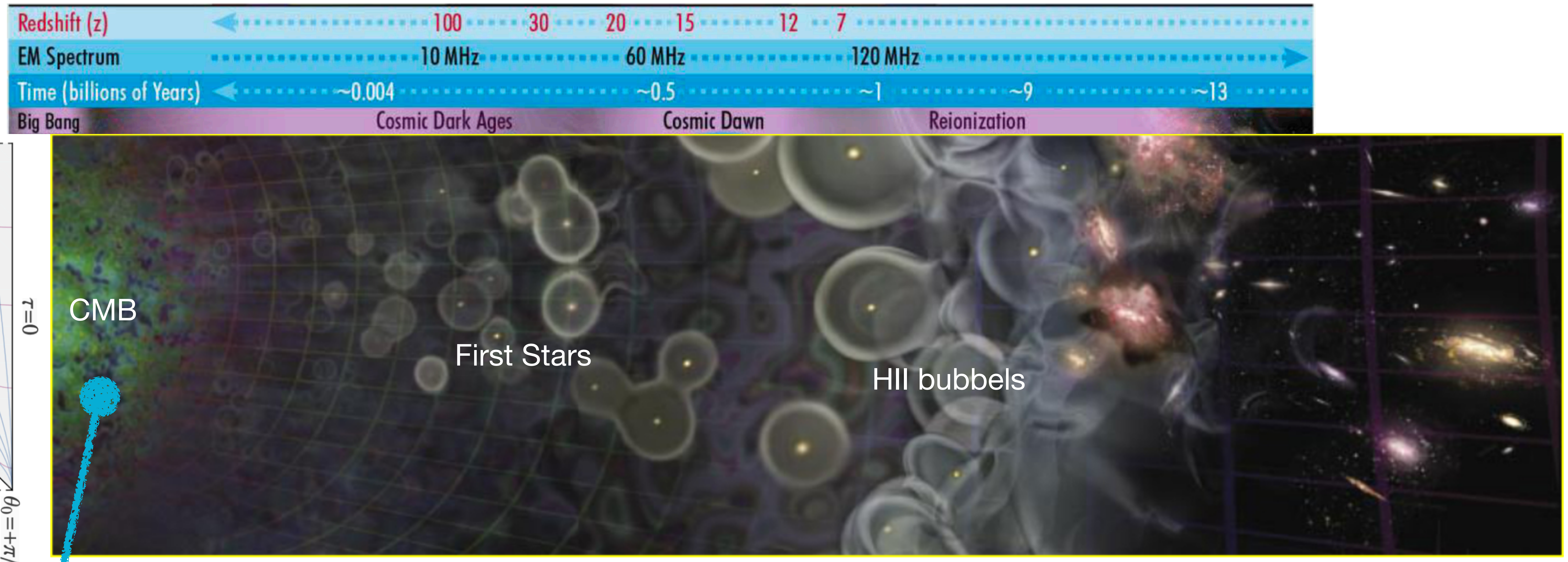

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New tracers asked!

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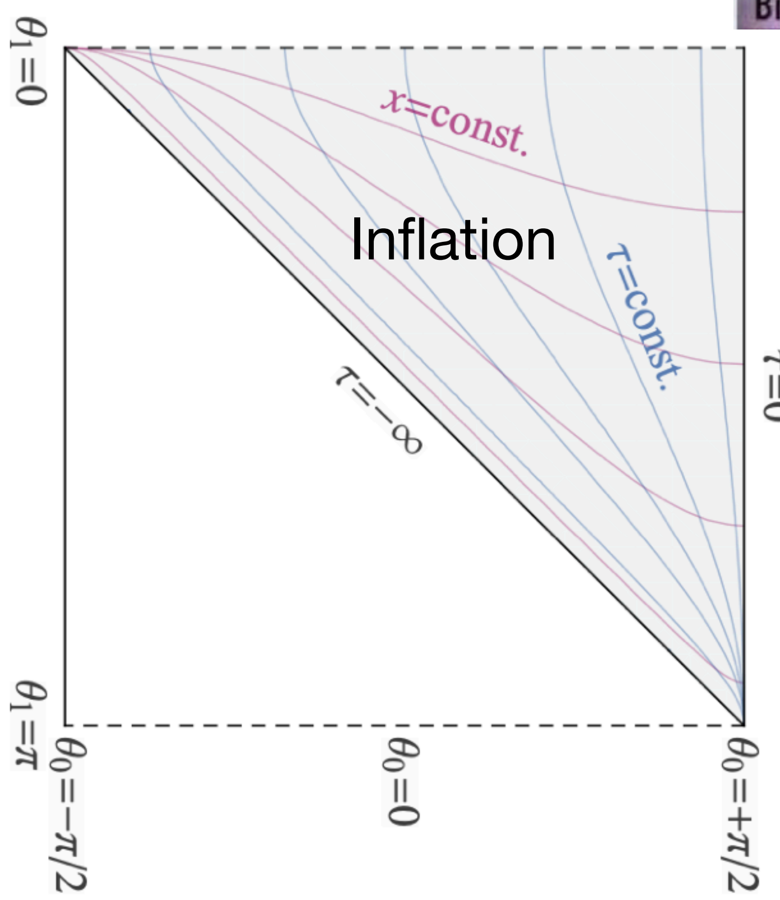
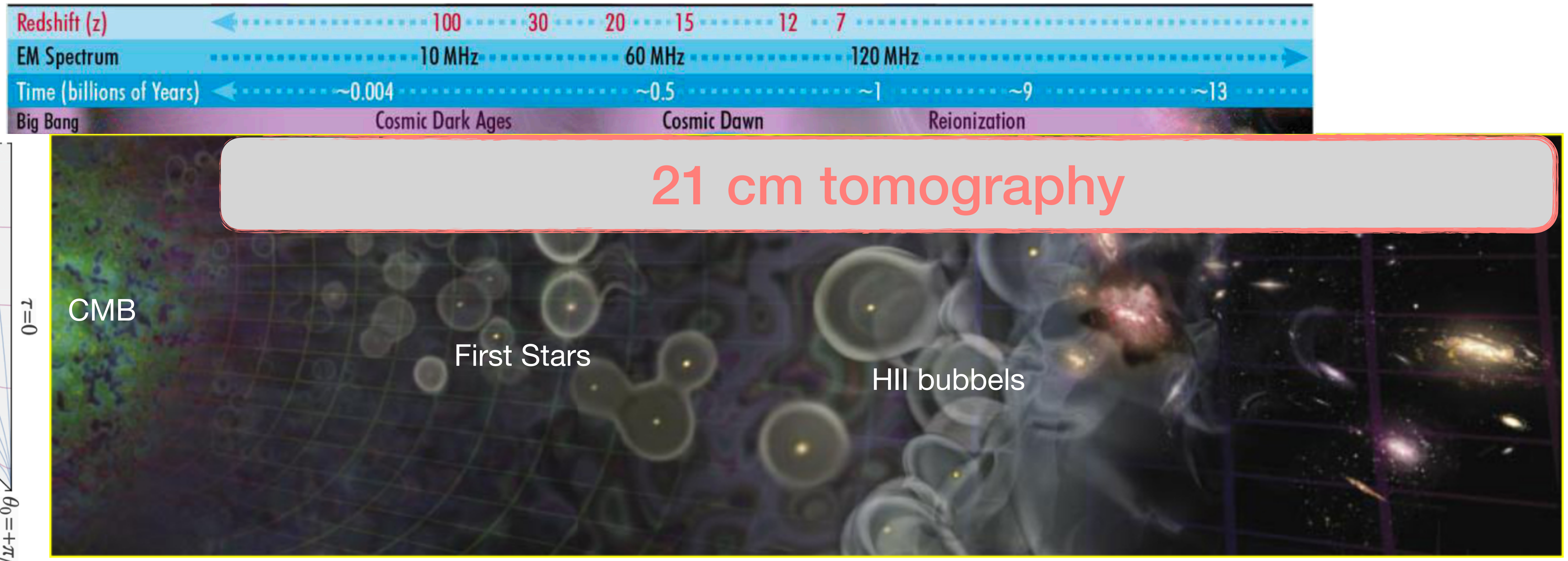
Background

Detect PNG with LSS

21cm is the most potential tracer of PNG.





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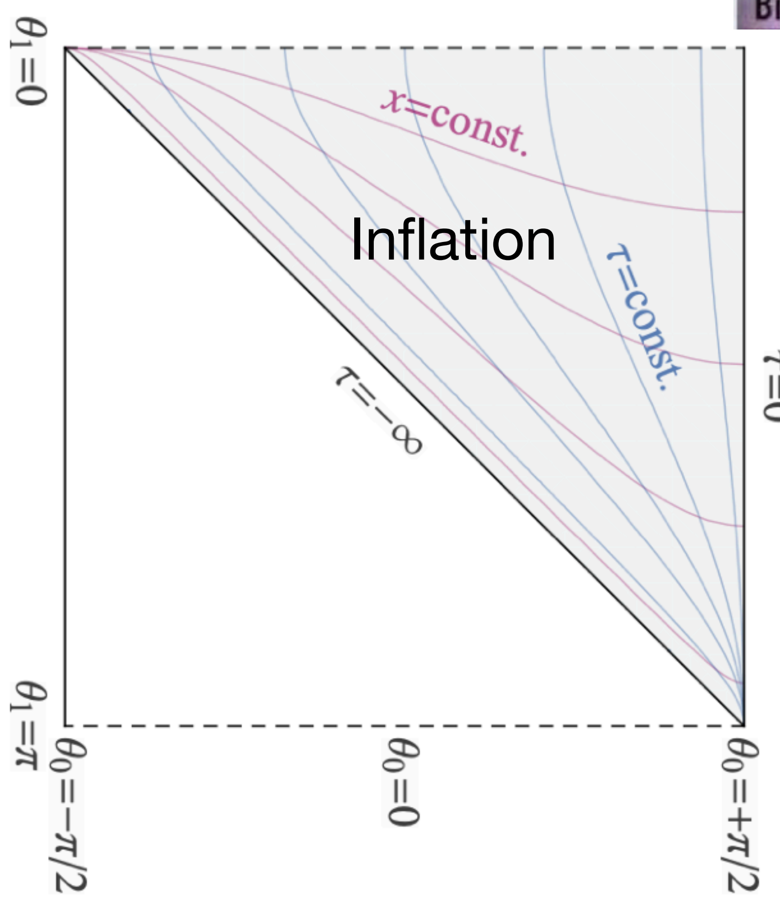

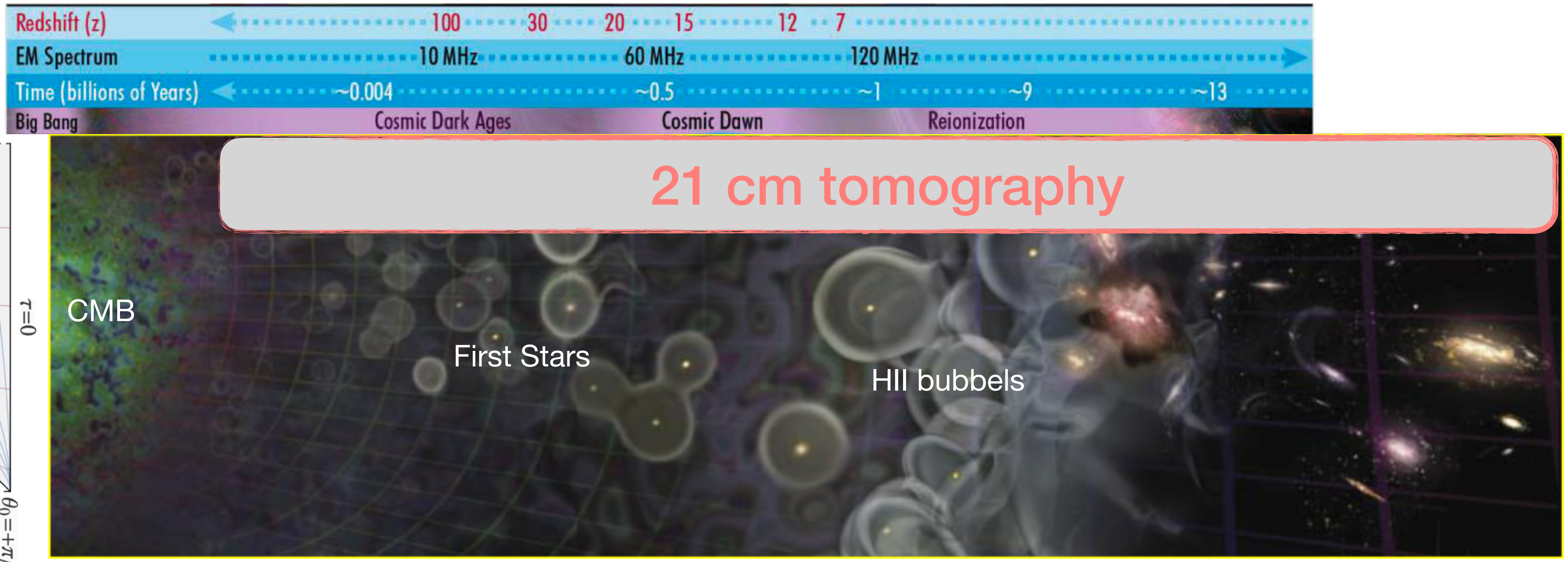

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scale-dependent bias

$$\Delta b(k,z) = 2 f_{\rm NL} \delta_{cr} (b_1(z) - 1) \mathscr{M}^{-1}(k,z)$$

Dalal et al. 2008

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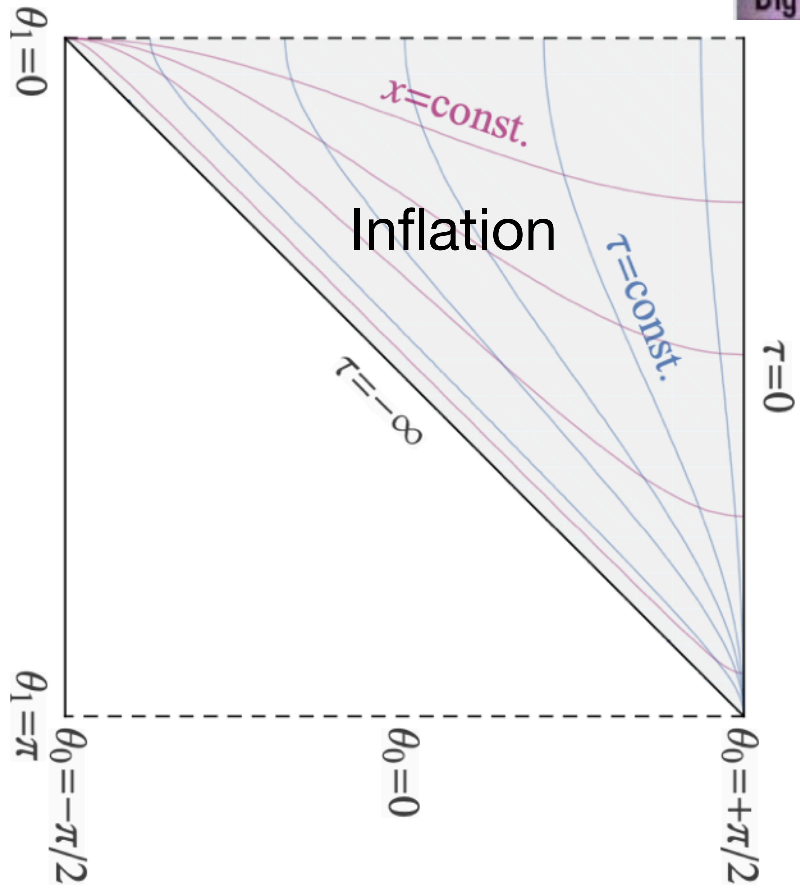

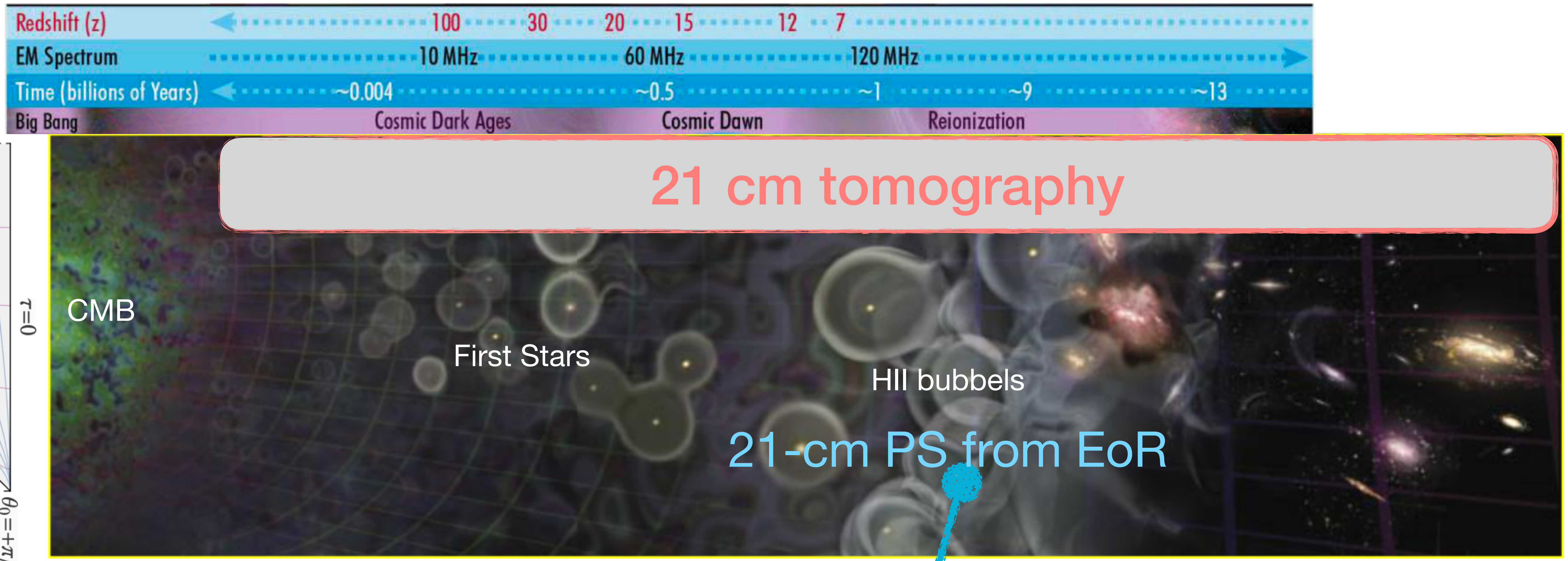

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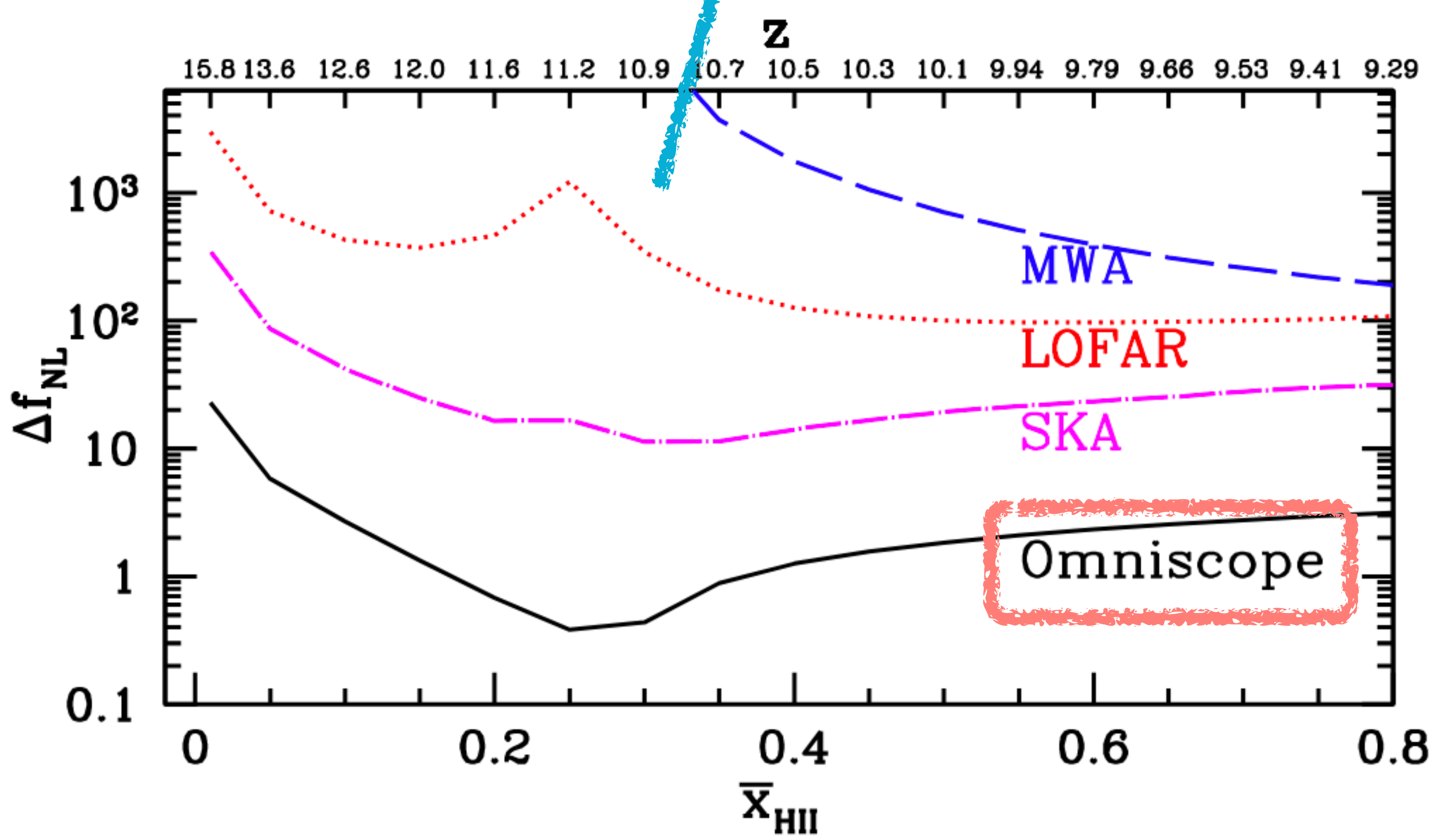

(Cosmic variance limited)

Mao et al. 2013

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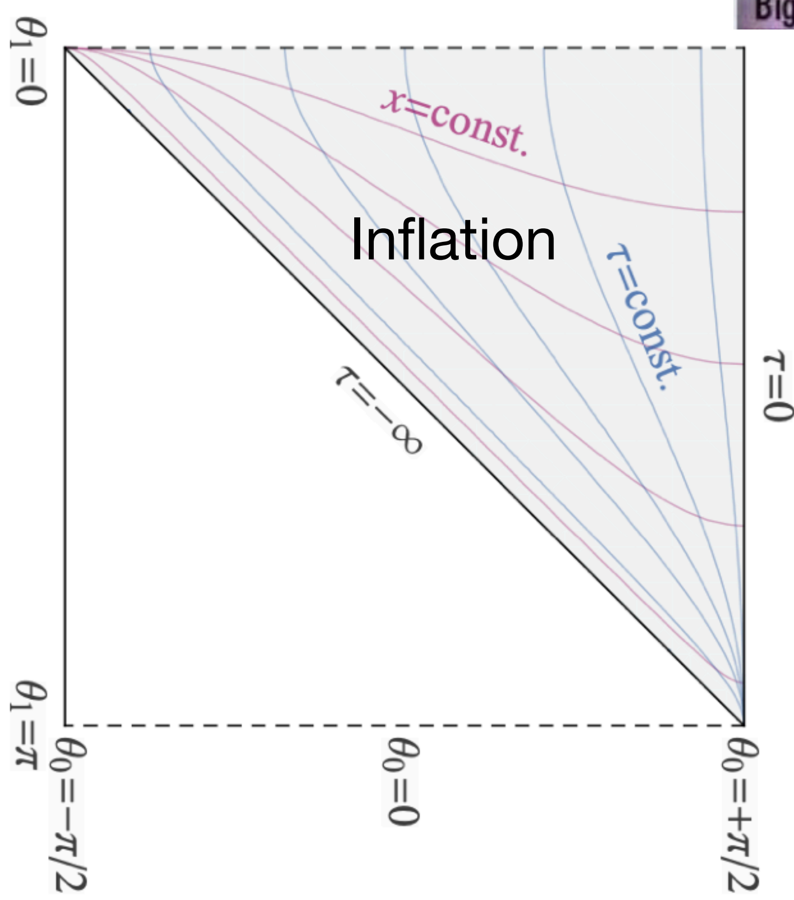

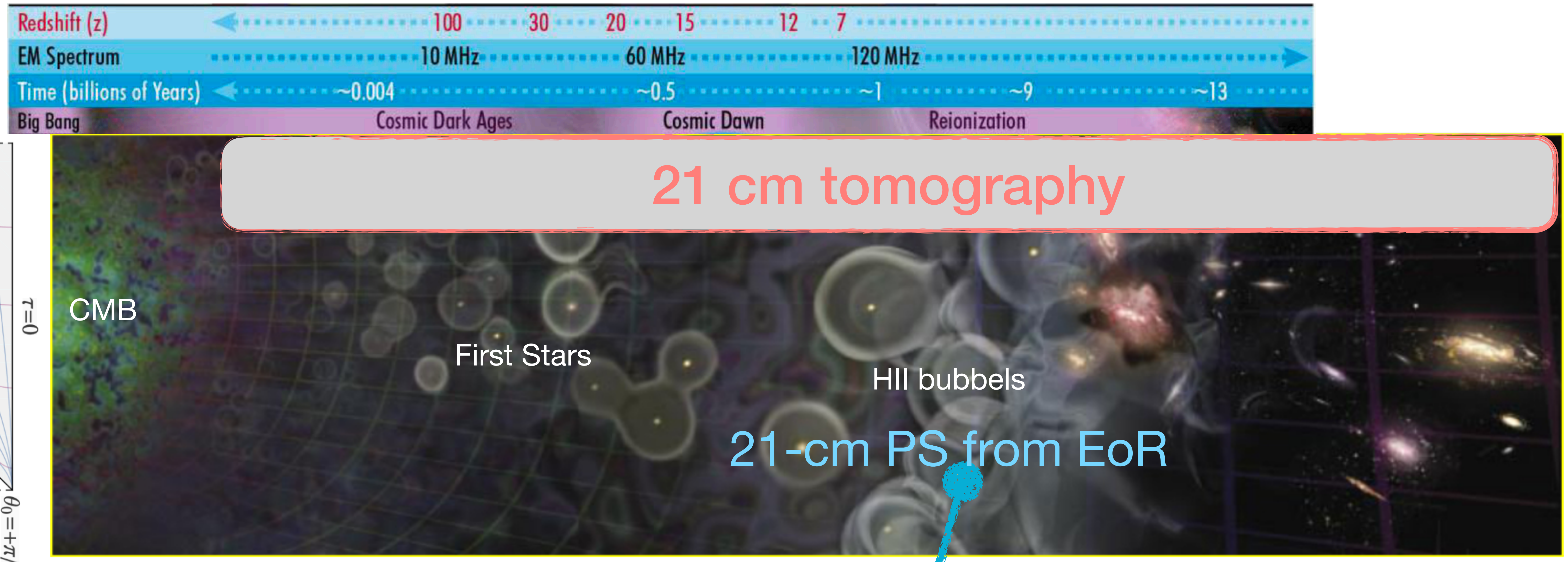

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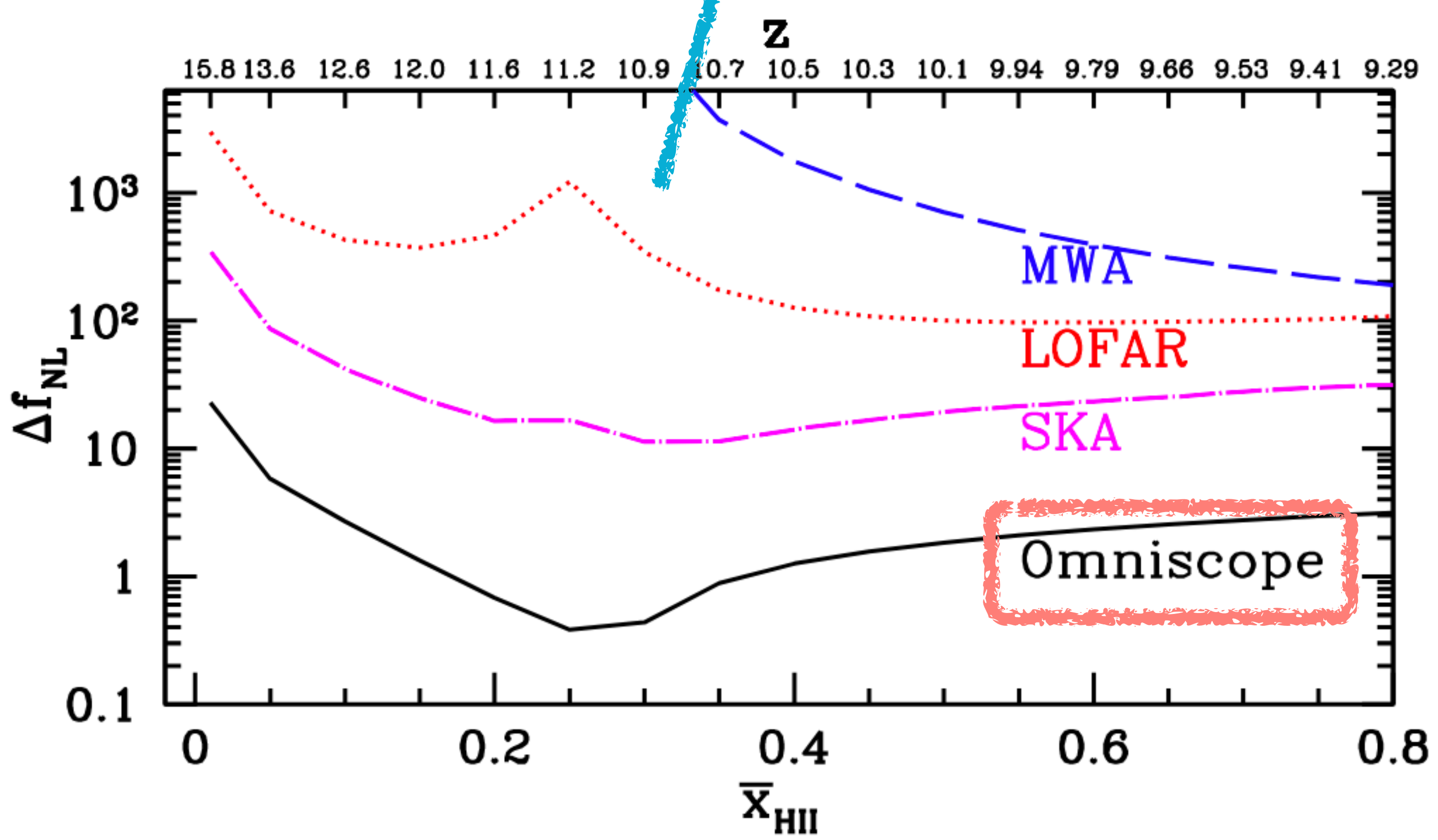


Mao et al. 2013

Still want better?

(Cosmic variance limited)

Methodology: Theory

21-cm Bispectrum

The quasi-linear model for RSD

peculiar velocity of the intergalactic gas → Redshift Space Distortion (RSD) effect of the 21 cm signal

$$B^{\mathrm{s,qlin}}_{\delta T_{\mathrm{b}}}(k_1,k_2,k_3) = \left(\widehat{\delta T_{\mathrm{b}}}(z_{\cos})\right)^3 \Big[B^{\mathrm{r}}_{\mathrm{HI,HI,HI}} + \left(\langle\mu_3^2\rangle B^{\mathrm{r}}_{\mathrm{HI,HI,H}} + 2\mathrm{perm.}\right) + \left(\langle\mu_1^2\mu_2^2\rangle B^{\mathrm{r}}_{\mathrm{H,H,HI}} + 2\mathrm{perm.}\right) + \langle\mu_1^2\mu_2^2\mu_3^2\rangle B^{\mathrm{r}}_{\mathrm{H,H,H}} \Big]$$

μ^0 terms: $B^{\mathrm{r}}_{\mathrm{HI,HI,HI}}$

μ^2 terms: $\left(\langle\mu_3^2\rangle B^{\mathrm{r}}_{\mathrm{HI,HI,H}} + 2\mathrm{perm.}\right)$

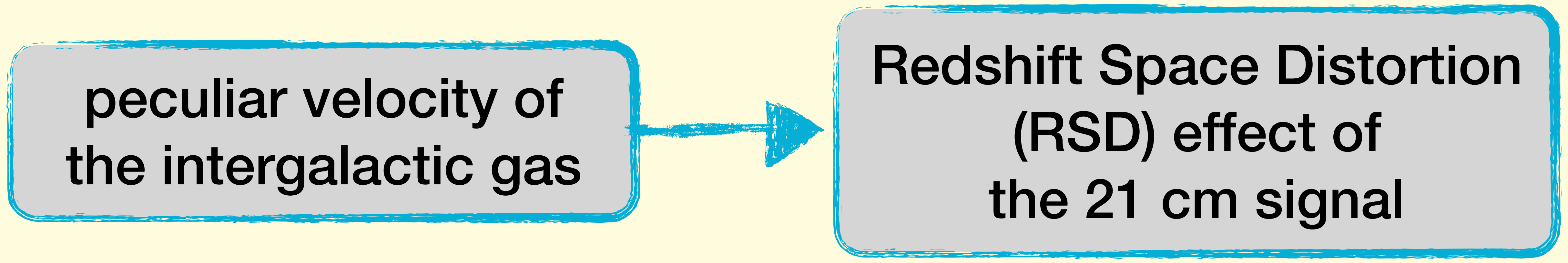
μ^4 terms: $\left(\langle\mu_1^2\mu_2^2\rangle B^{\mathrm{r}}_{\mathrm{H,H,HI}} + 2\mathrm{perm.}\right)$

μ^6 terms: $\langle\mu_1^2\mu_2^2\mu_3^2\rangle B^{\mathrm{r}}_{\mathrm{H,H,H}}$

Methodology: Theory

21-cm Bispectrum

The quasi-linear model for RSD



μ^0 terms μ^2 terms

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$x_{\mathrm{HI}} \gtrsim 0.5$ μ^4 terms μ^6 terms

Methodology: Theory

21-cm Bispectrum

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μ^0 terms

μ^2 terms

μ^4 terms

μ^6 terms

$x_{\mathrm{HI}} \gtrsim 0.5$

21-cm Bispectrum

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μ^0 terms μ^2 terms

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$x_{\rm HI} \gtrsim 0.5$ μ^4 terms μ^6 terms

$$b_1^3 B^{\rm LO}_{mmm} + \left[b_1^2 b_2 P_{\rm L}(k_1) P_{\rm L}(k_2) + 2\text{ perm.}\right]$$

$$B_{\rm HI,HI,HI} = B^{\rm G}_{\rm HI,HI,HI} + b_1^3 B^{(1)}_{mmm} + \Big(P_L(k_1)P_L(k_2) \Big\{ b_1^2\Big[\mathcal{R}_b\left(\Delta b(k_1)+\Delta b(k_2)\right) + \mu_{12}\left(\frac{k_1}{k_2}\Delta b(k_1) + \frac{k_2}{k_1}\Delta b(k_2)\right)\Big] + b_1\left(2F_2(\mathbf{k}_1,\mathbf{k}_2)b_1 + b_2\right)\left(\Delta b(k_1)+\Delta b(k_2)\right)\Big\} + 2\text{ perm.}\Big)$$

$$B^{(1)}_{mmm} = 2f_{\rm NL}\left[P_L(k_1)P_L(k_2)\mathcal{M}(k_3)\mathcal{M}^{-1}(k_1)\mathcal{M}^{-1}(k_2) + 2\text{ perm.}\right]$$

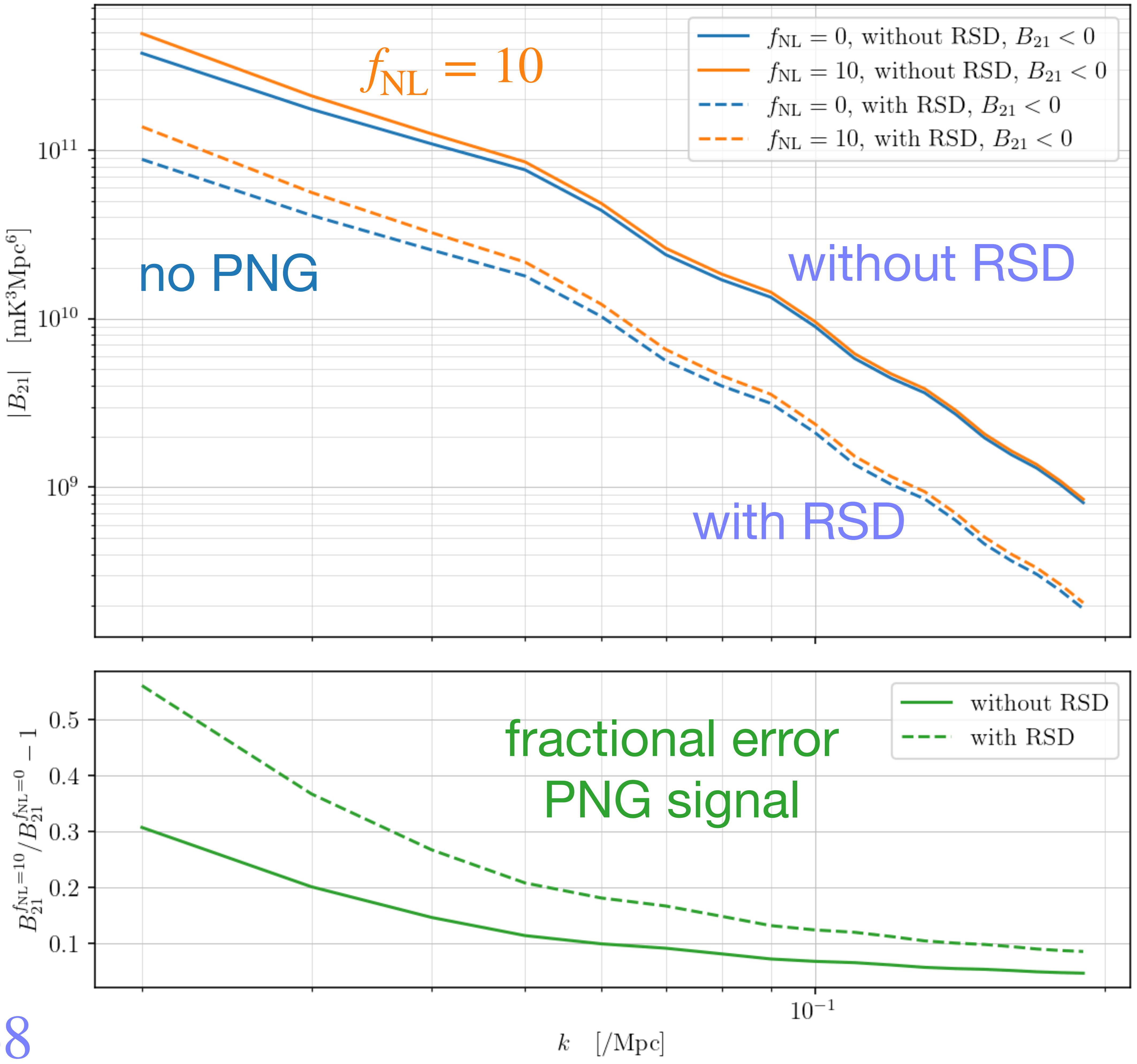
- Nonlinear gravitational evolution
- Nonlinear bias (second order bias)
- Linear growth of PNG
- PNG effect on bias

Results: Theory

Predictions of the Signal

- Fiducial Model
  - $T_{\rm vir} = 50000\mathrm{K}$
  - $\zeta = 50$

Absolute value of bispectrum



Size of the triangle

$z = 10.5$
$x_{\rm HI} = 0.83$
$b_1 = -0.68$
$b_2 = -12.9$

Zhao, Mao & Wang, in prep.

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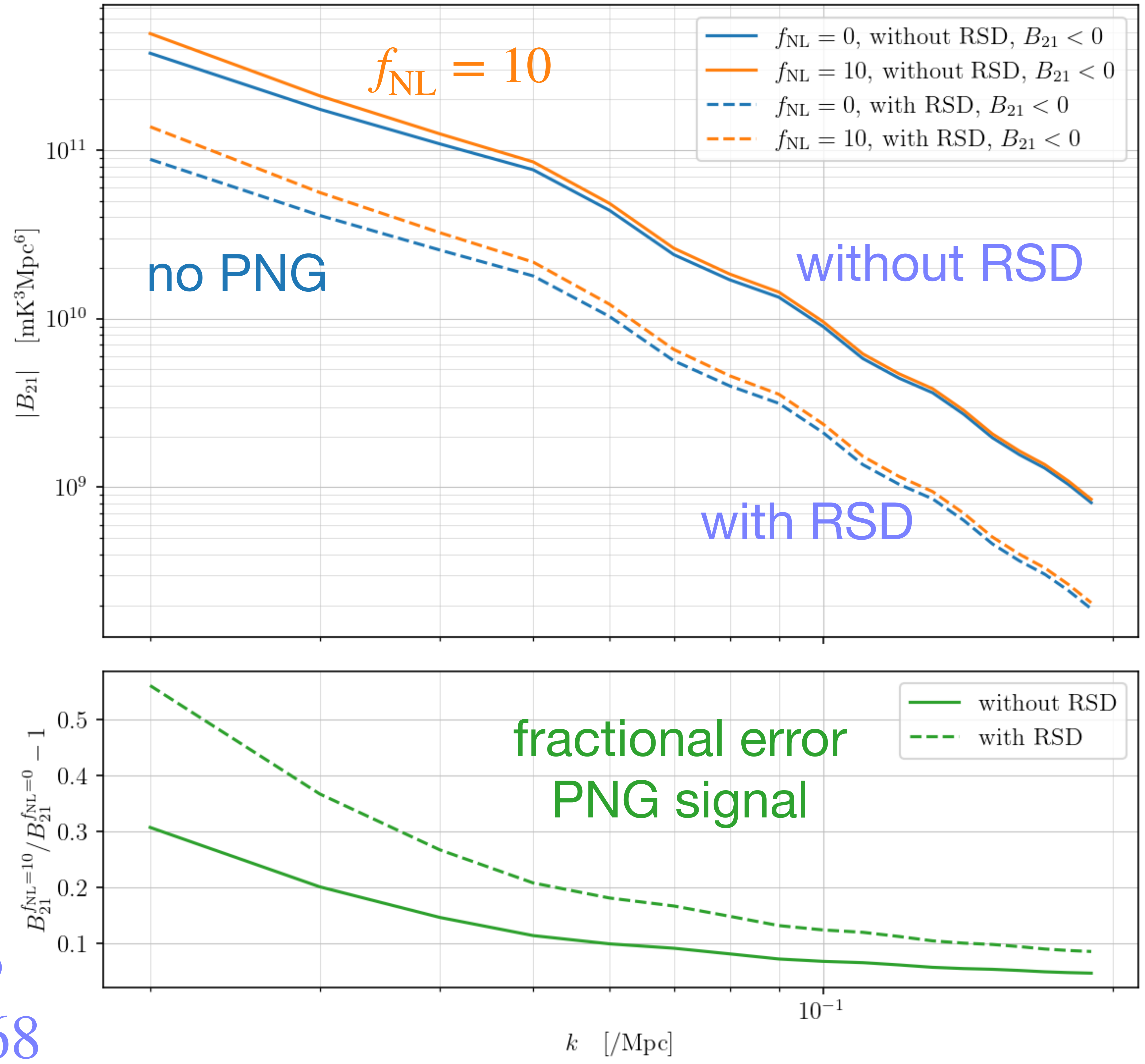

Size of the triangle

$\triangle$

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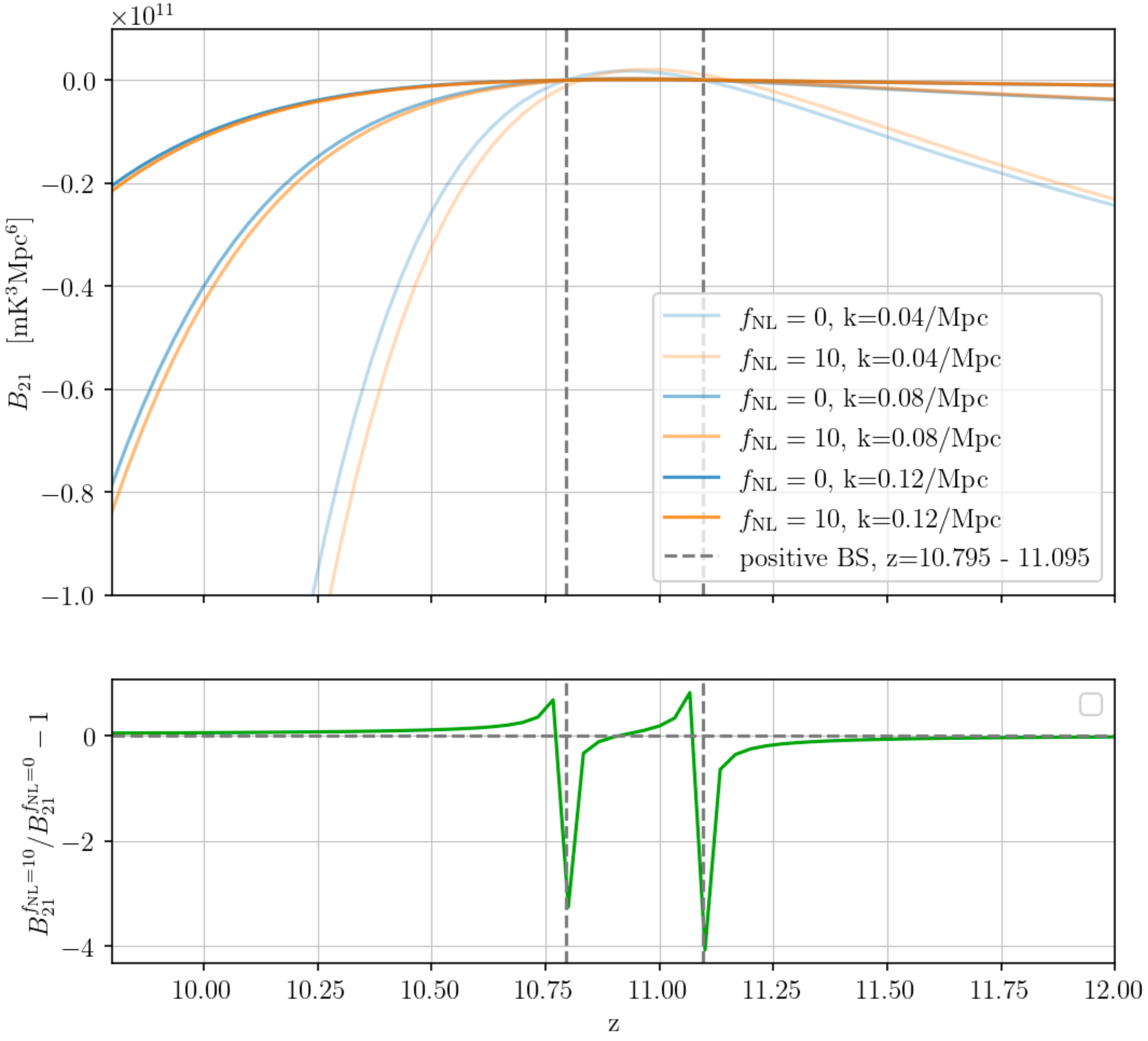

redshift

Zhao, Mao & Wang, in prep.

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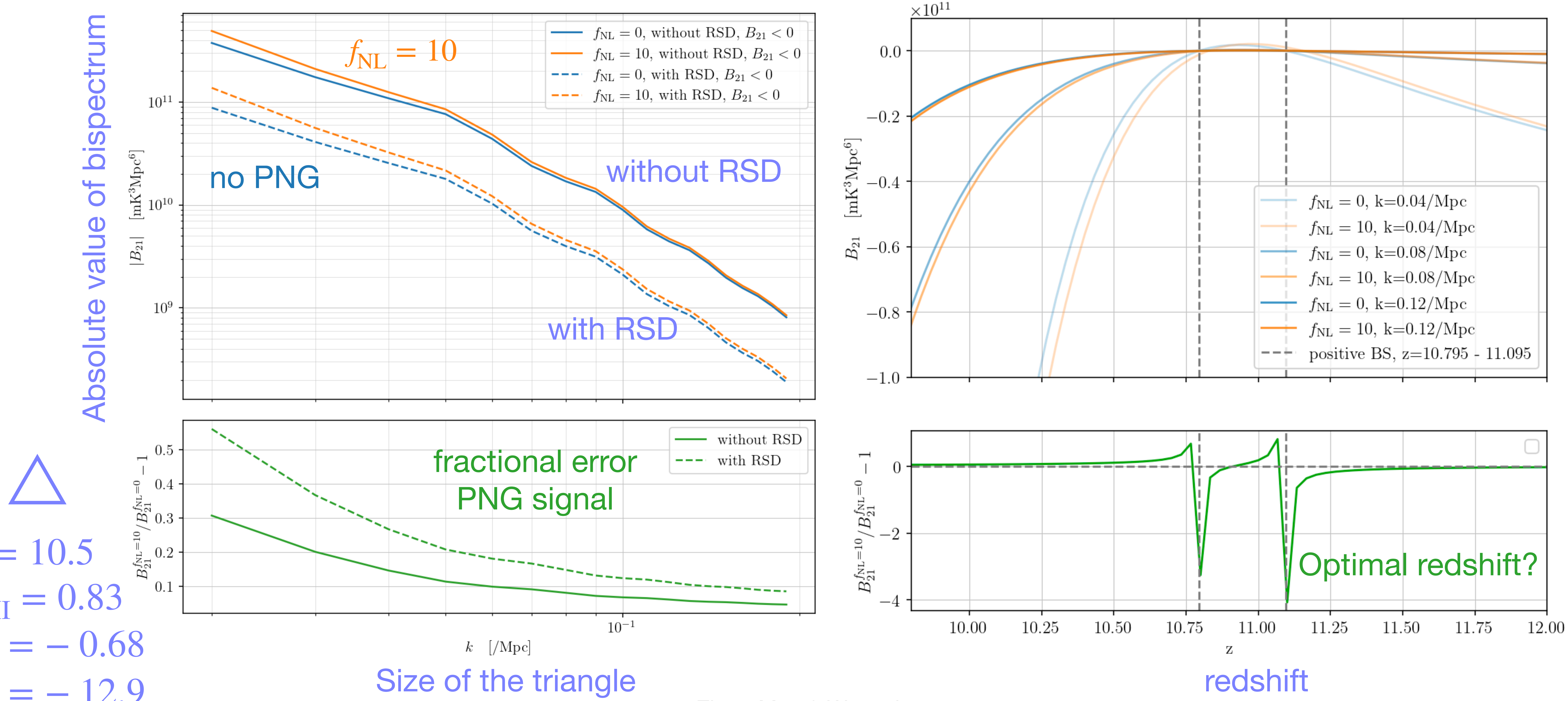

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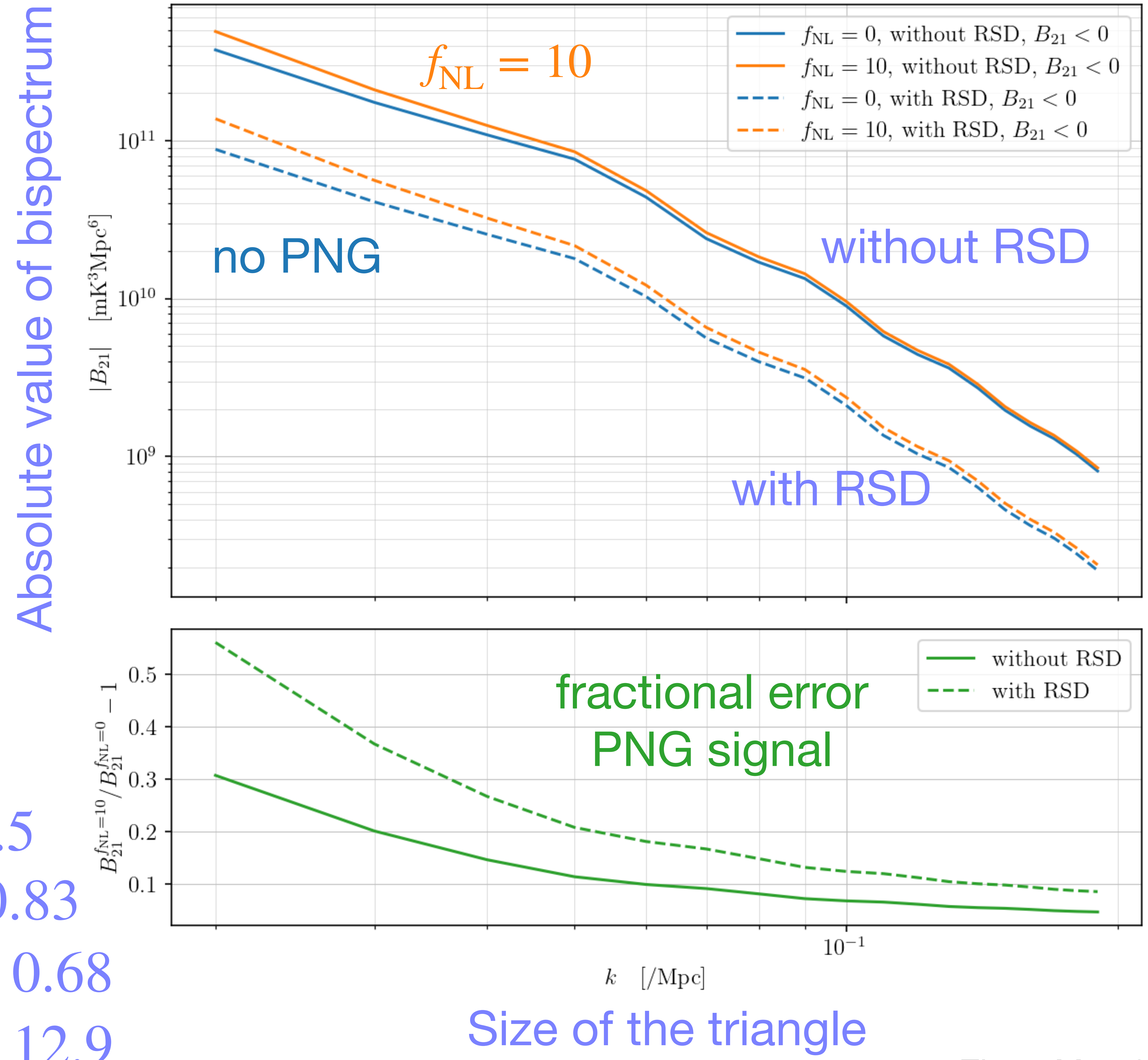

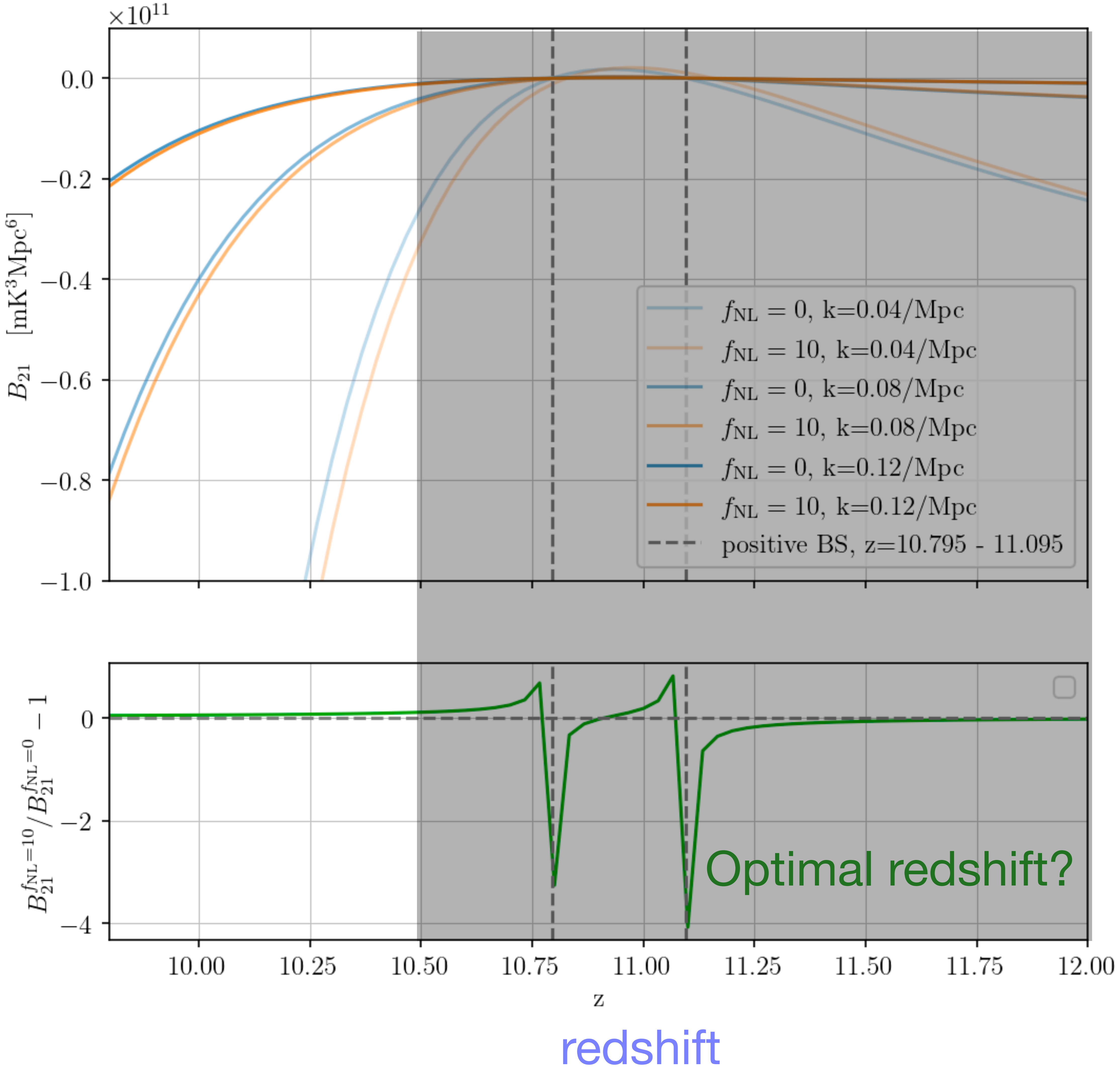

△

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Zhao, Mao & Wang, in prep.

Methodology: Forecast

Forecast the Observability

Forecast the Observability

Fisher Formalism

$$F_{\alpha\beta} \approx \sum_i F_{\alpha\beta}^{(i)} \qquad F_{\alpha\beta}^{(i)} \approx F_{\alpha\beta}^{P,(i)} + F_{\alpha\beta}^{B,(i)}$$

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Forecast the Observability

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Noise

- Cosmic Variance + Thermal Noise
- Foreground removed with $k_{\parallel,\,\mathrm{min}} = 2\pi/(yB)$ —— “EoR window”

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Experiment Settings

Experiment	$N_{\rm in}$	$L_{\rm min}$(m)	A_e [m^2]	Ω[sr]$^{\rm a}$
SKA2-LOW	896	40	$819.2\frac{(110{\rm MHz})^2}{f^2}$	λ^2/A_e
Omniscope	10^6	1	1	2π

- bandwidth = 8MHz
- integral time = 4000h
- k_max = 0.15 /Mpc

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a Fast Fourier Telescope, Tegmark & Zaldarriaga, 2009

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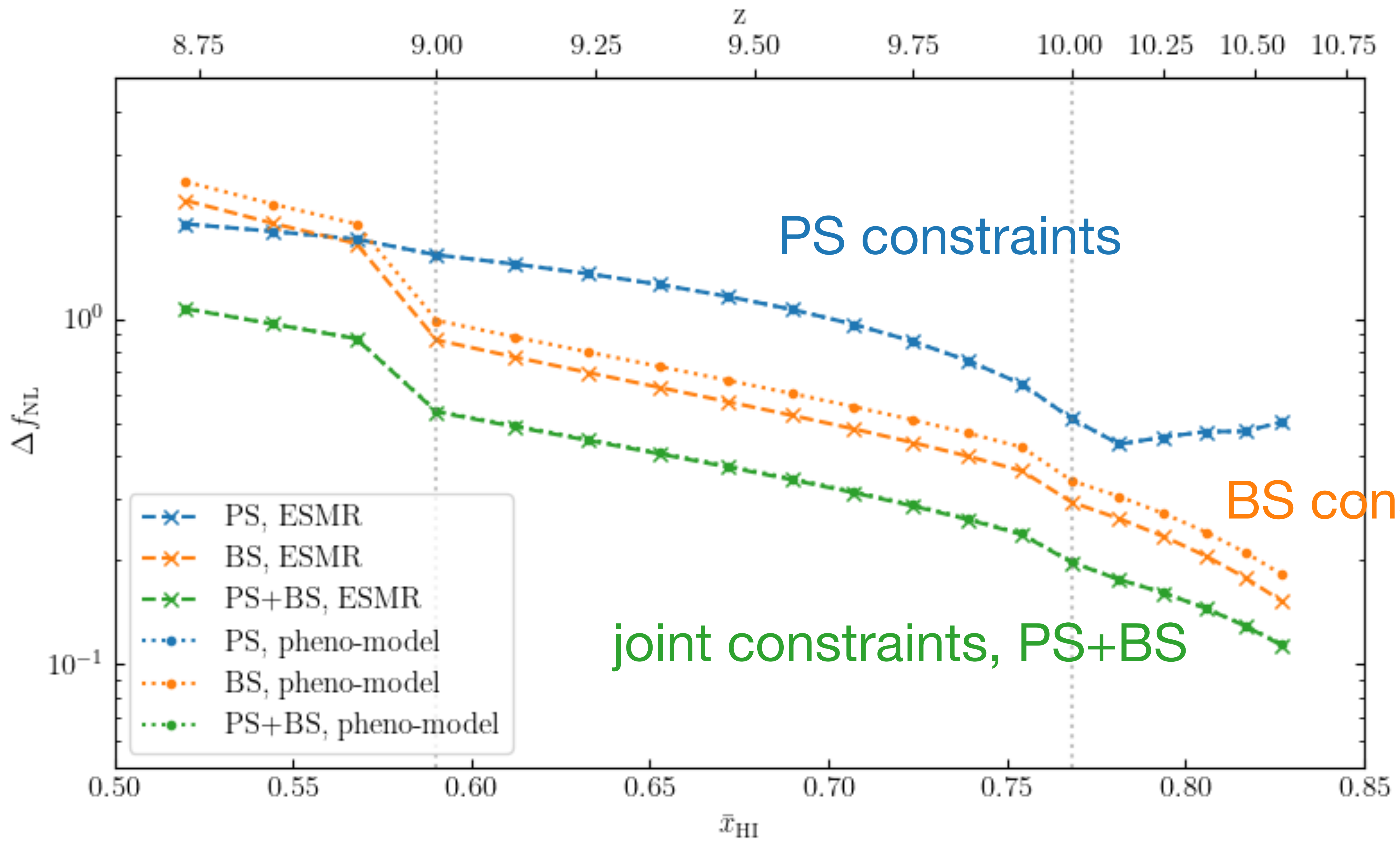
Results

Single Epoch Constraints

1-sigma error constraint of fNL



Zhao, Mao & Wang, in prep.

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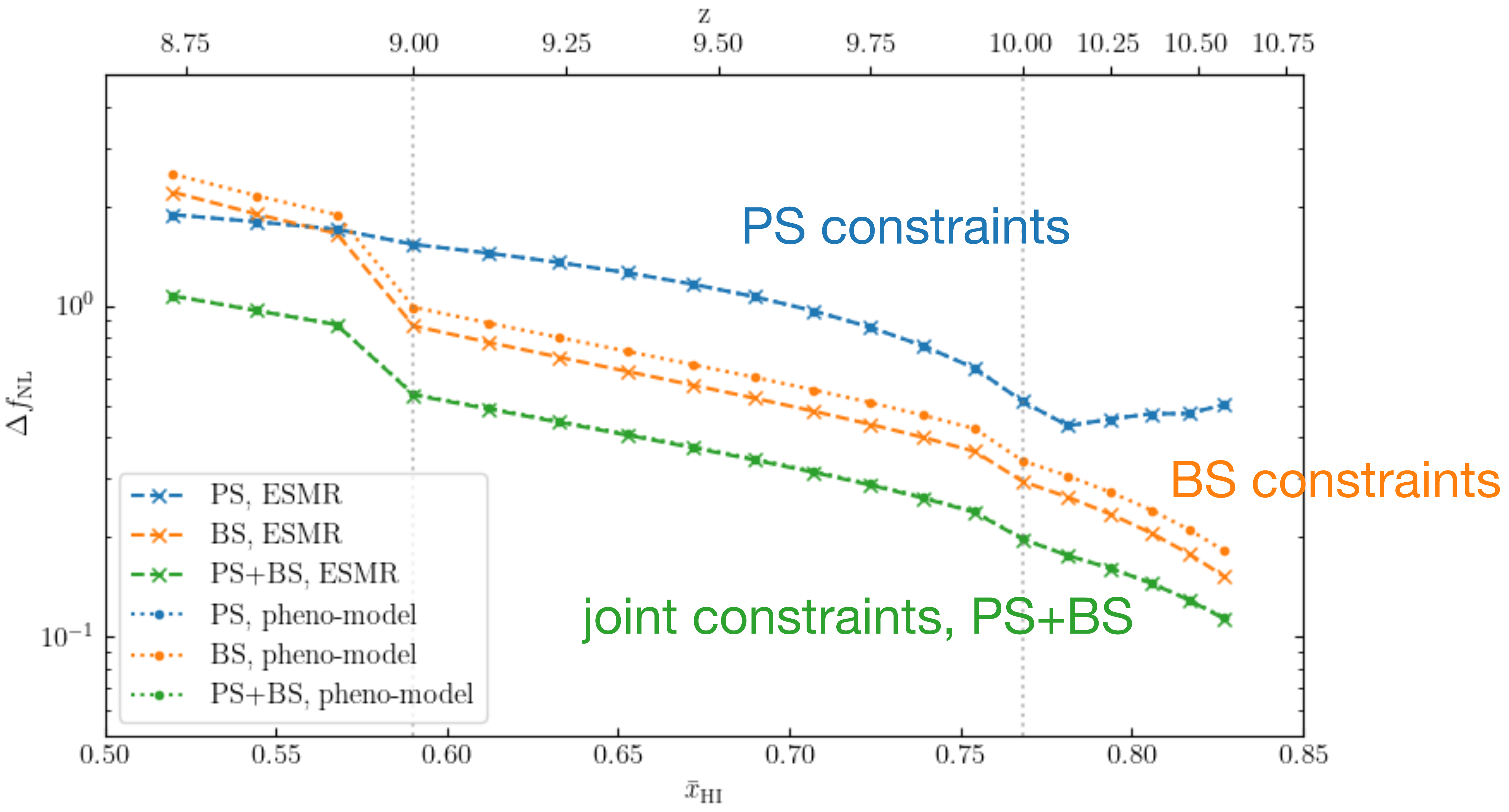
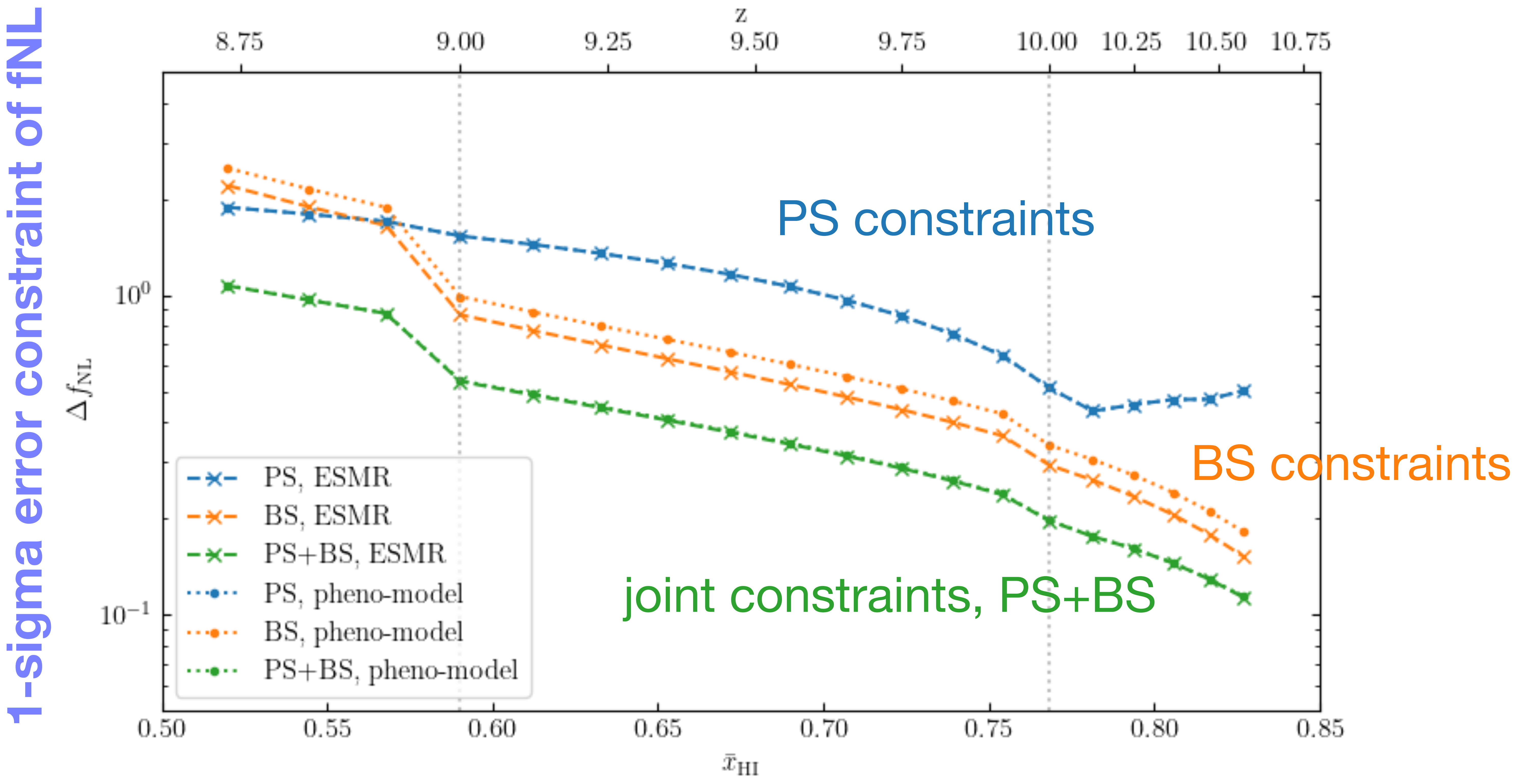


Zhao, Mao & Wang, in prep.

Single Epoch Constraints

For Omniscope, BS constraints are more stringent than PS at high-z.



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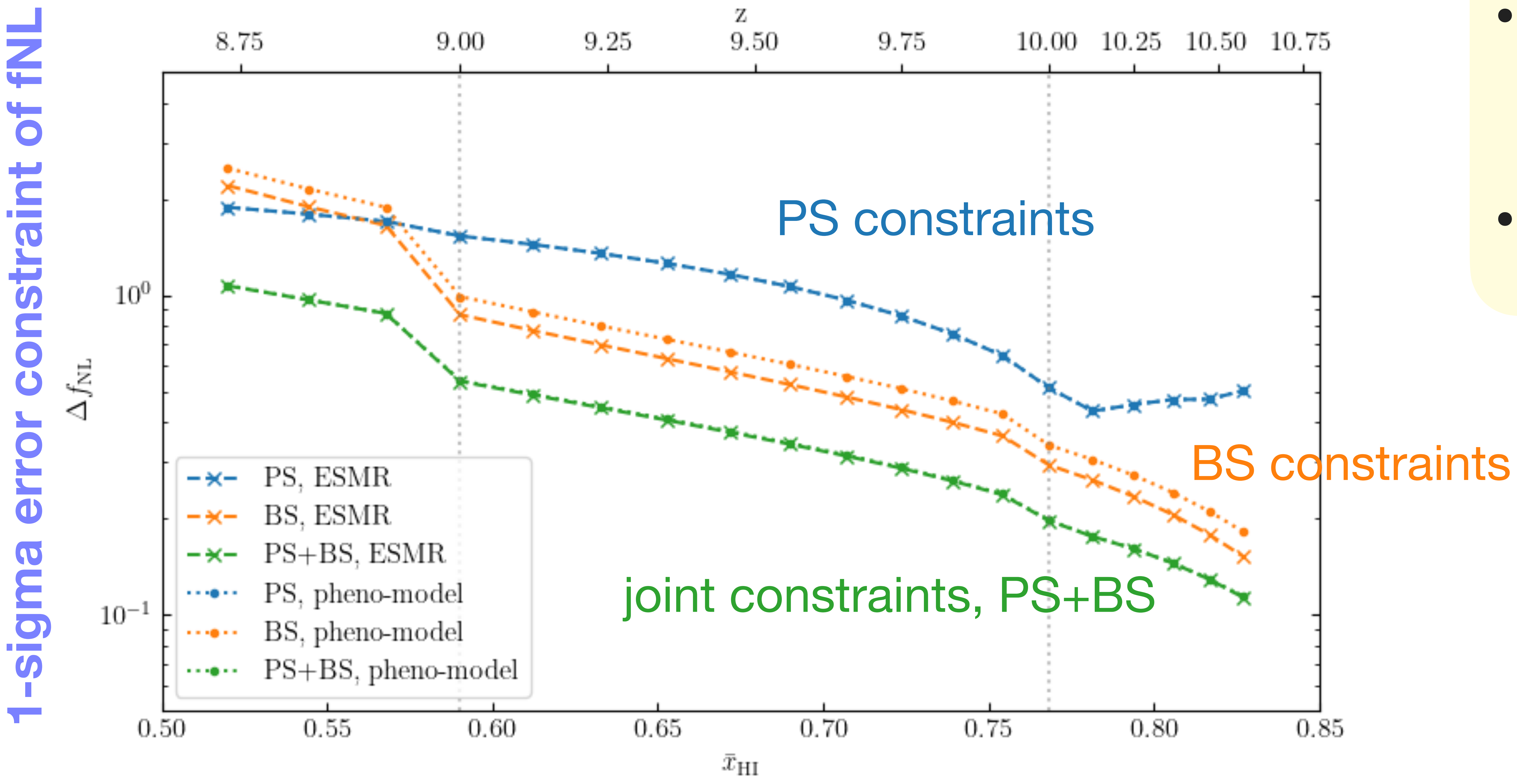

Zhao, Mao & Wang, in prep.

- Phenomenological ("pheno"-) model: $\lambda_\alpha = \{f_{\rm NL}, x_{\rm HI}(z_j), b_1(z_j), b_2(z_j)\}$
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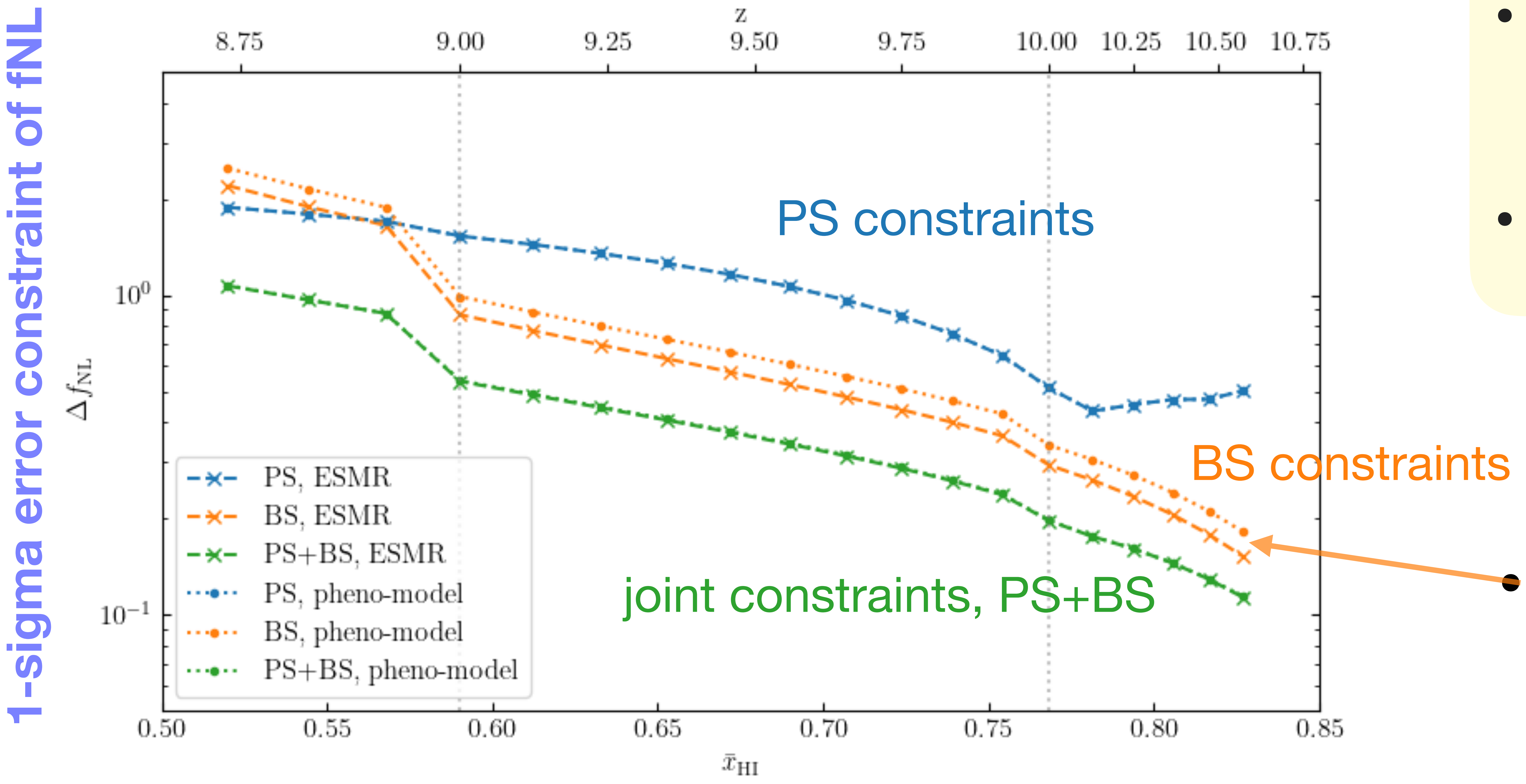

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- ESMR is slightly better than pheno-model for BS.

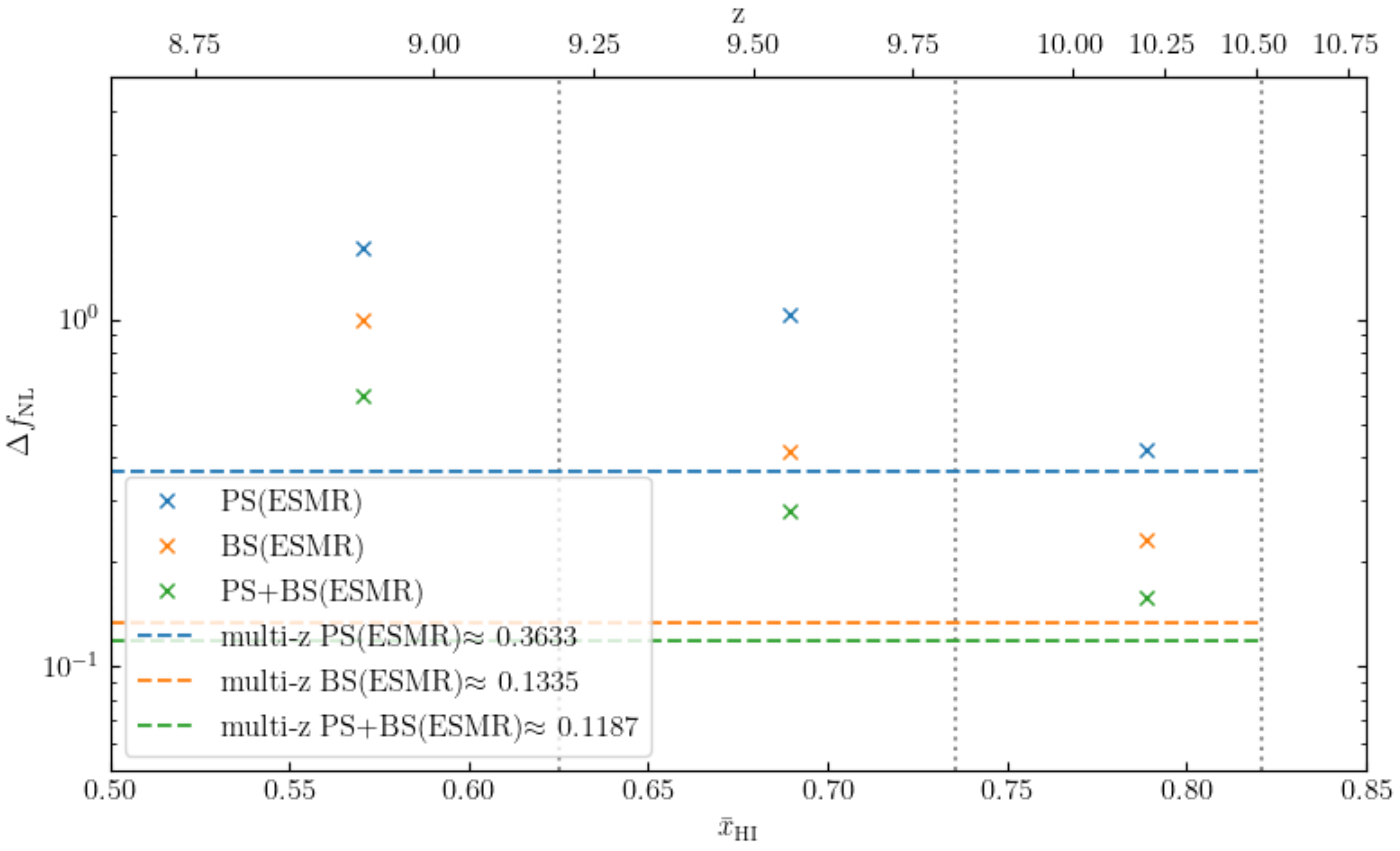
Results

Multi-Epoch Constraints

BS helps improving the PS constraints.

- Omniscope

1-sigma error constraint of fNL



Zhao, Mao & Wang, in prep.

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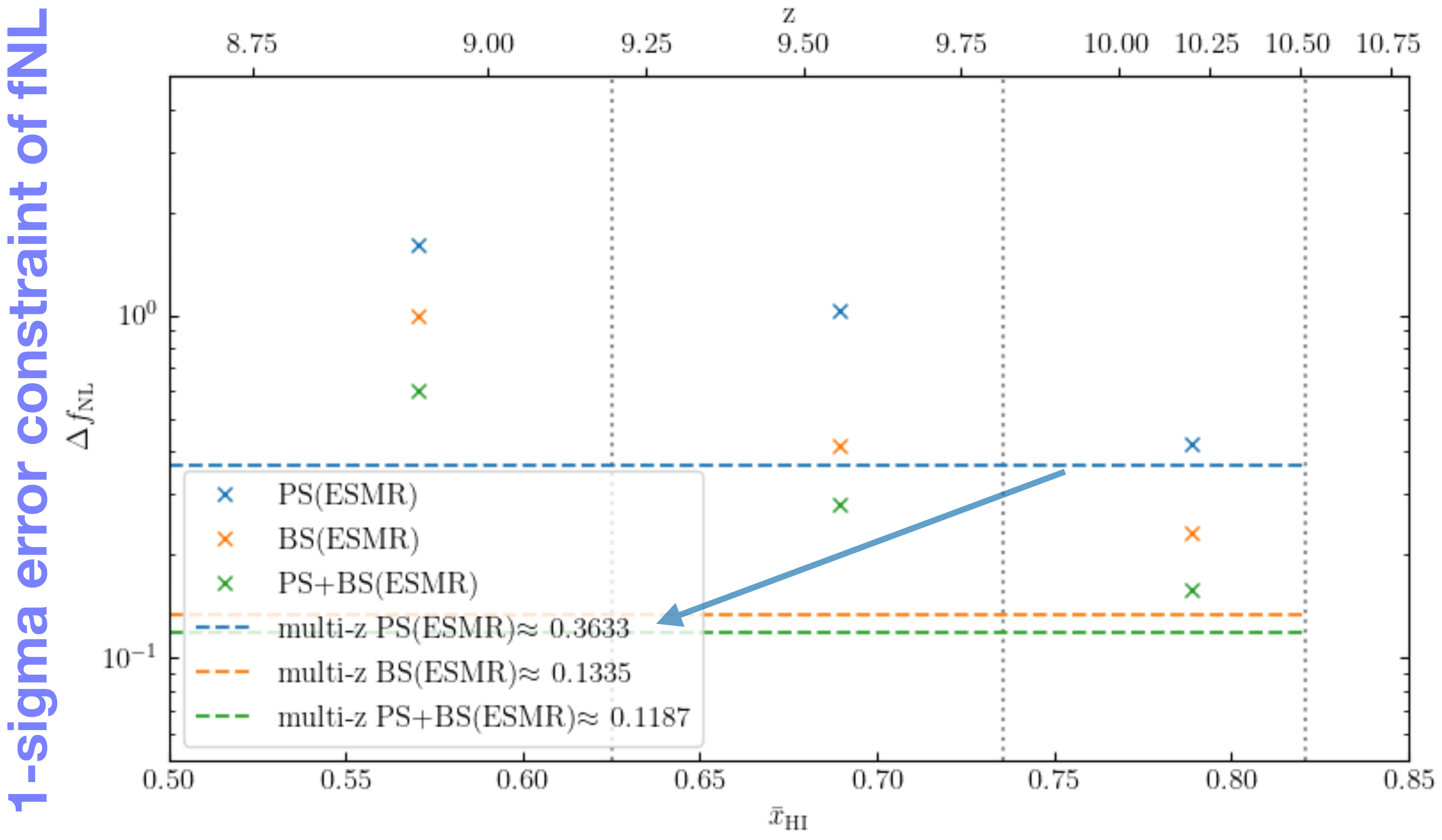

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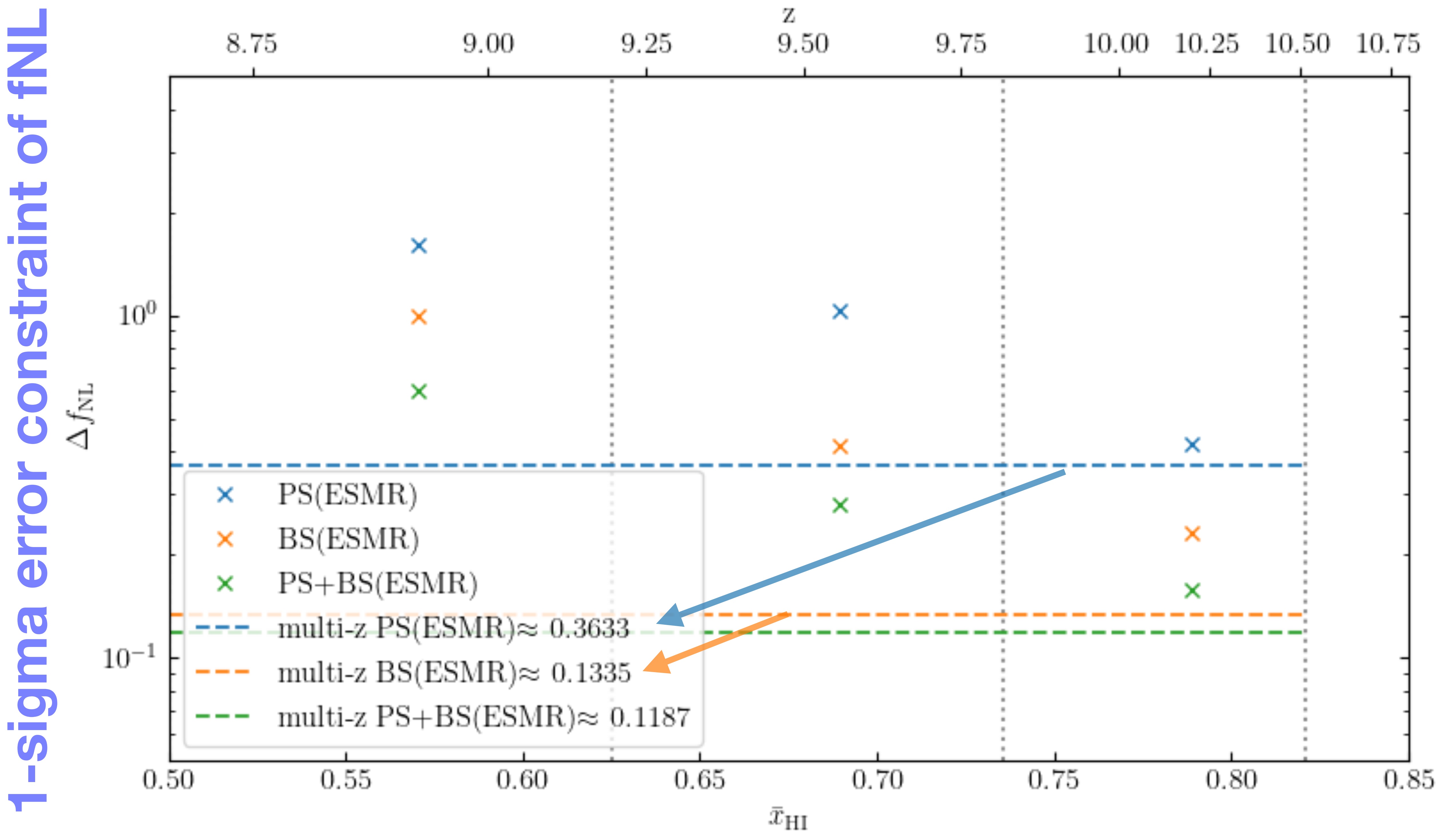

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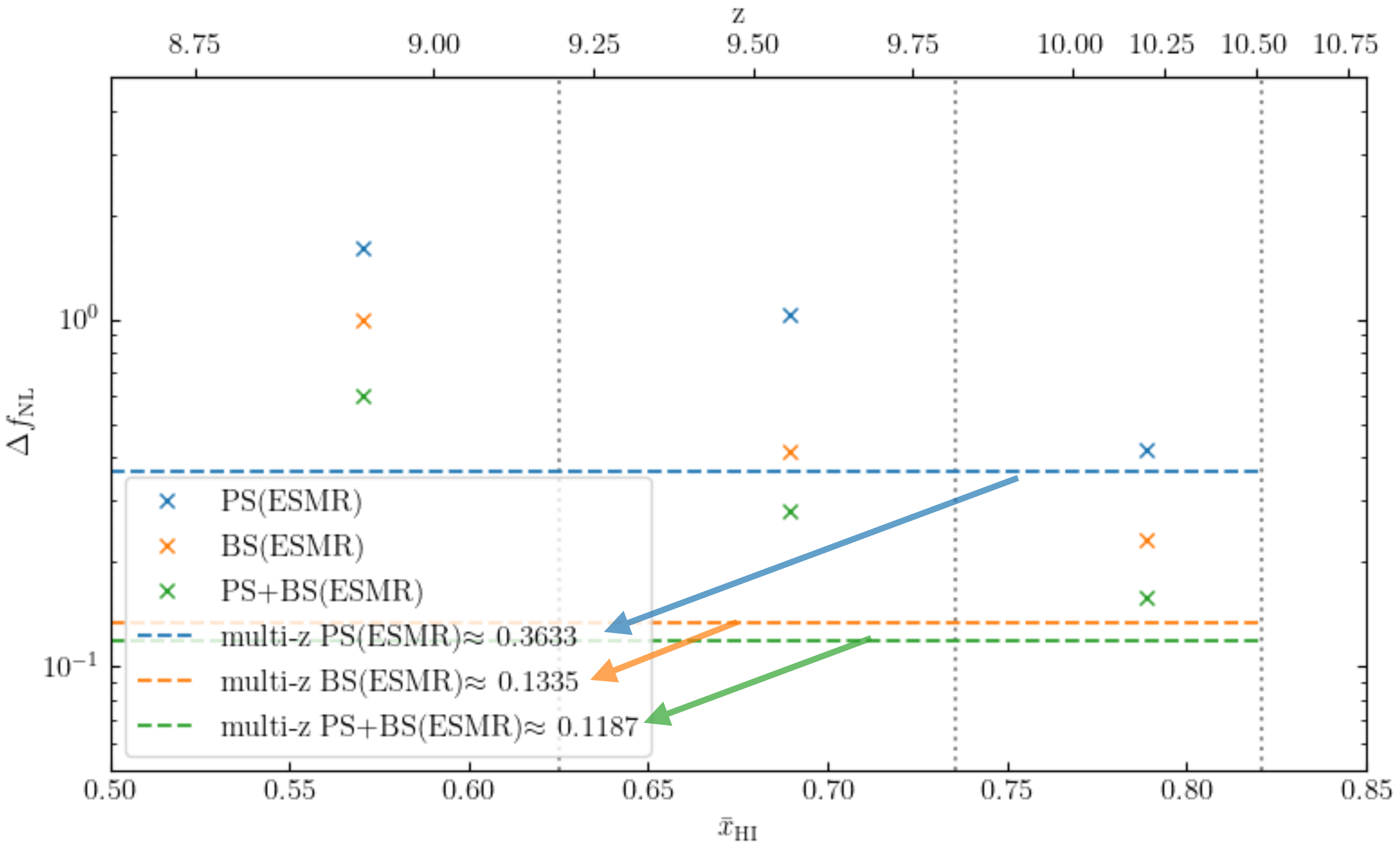

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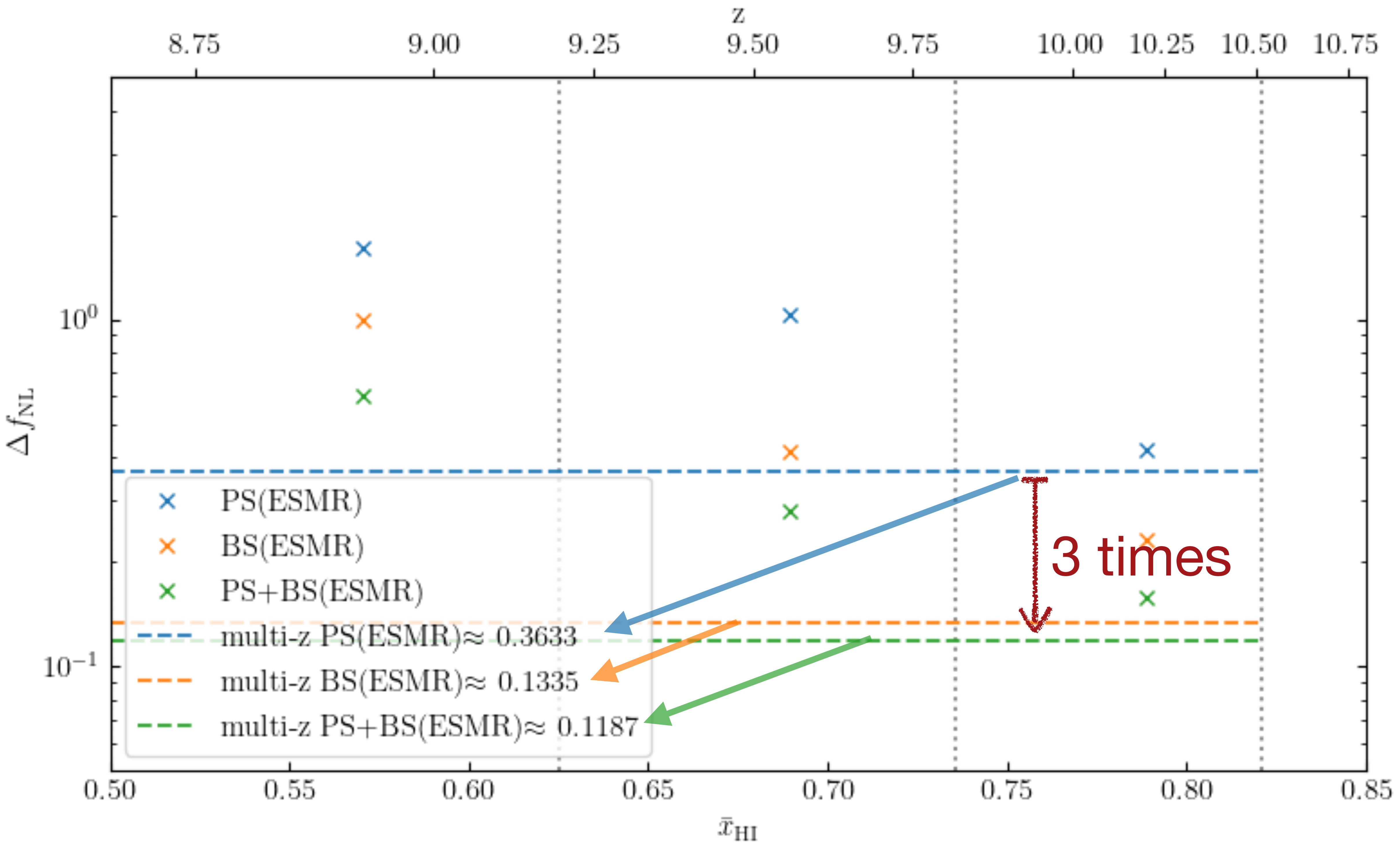

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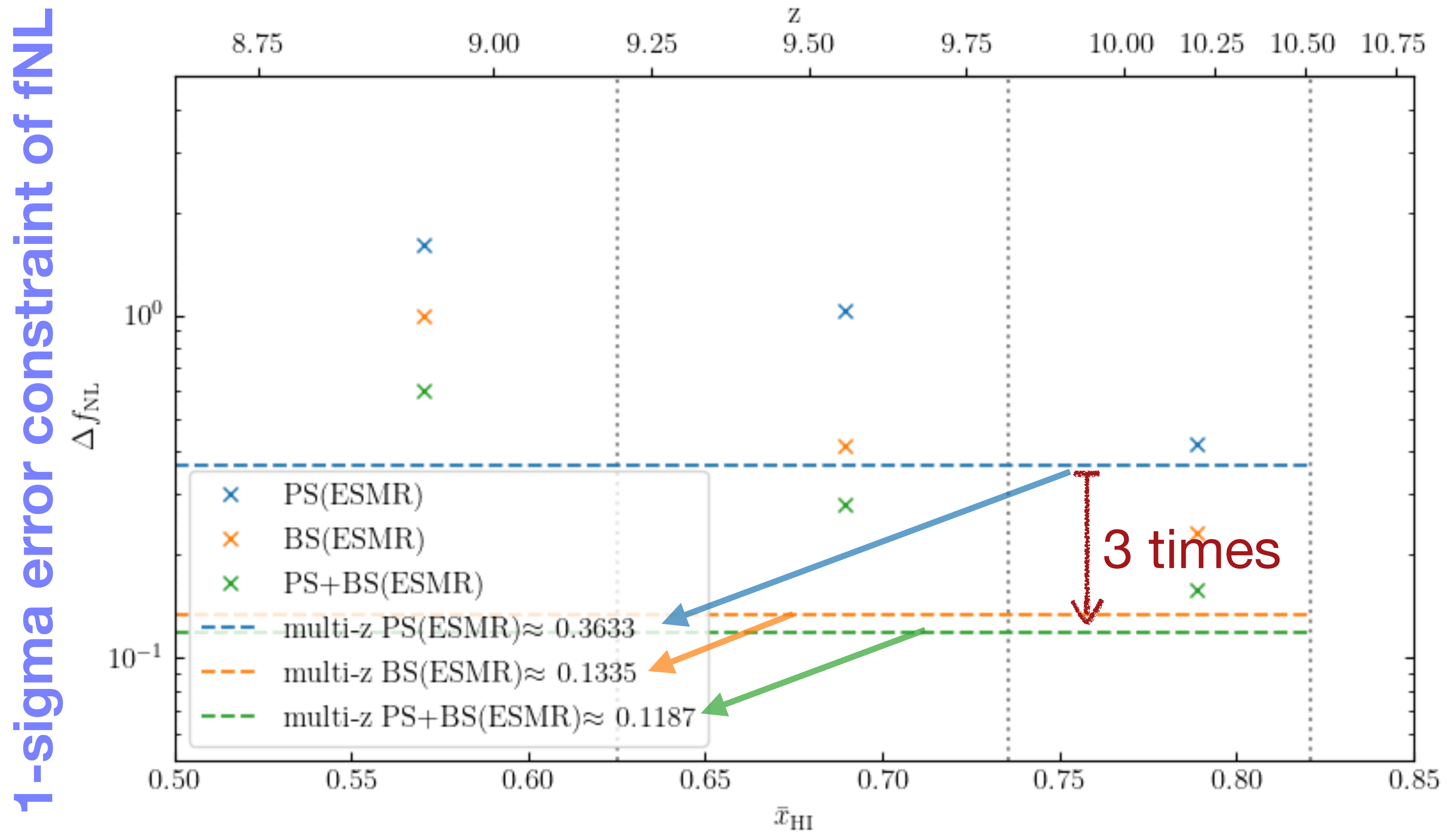

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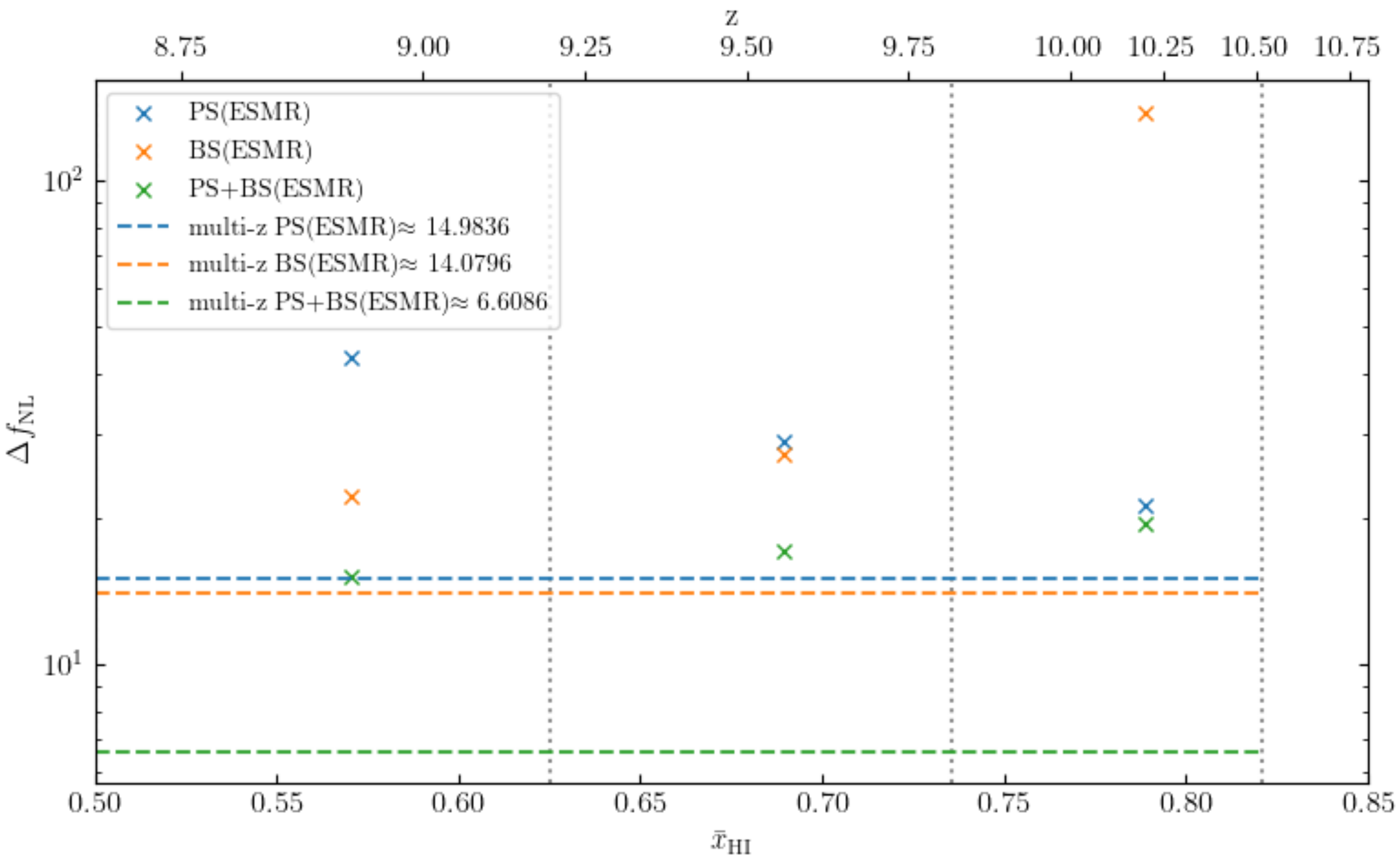

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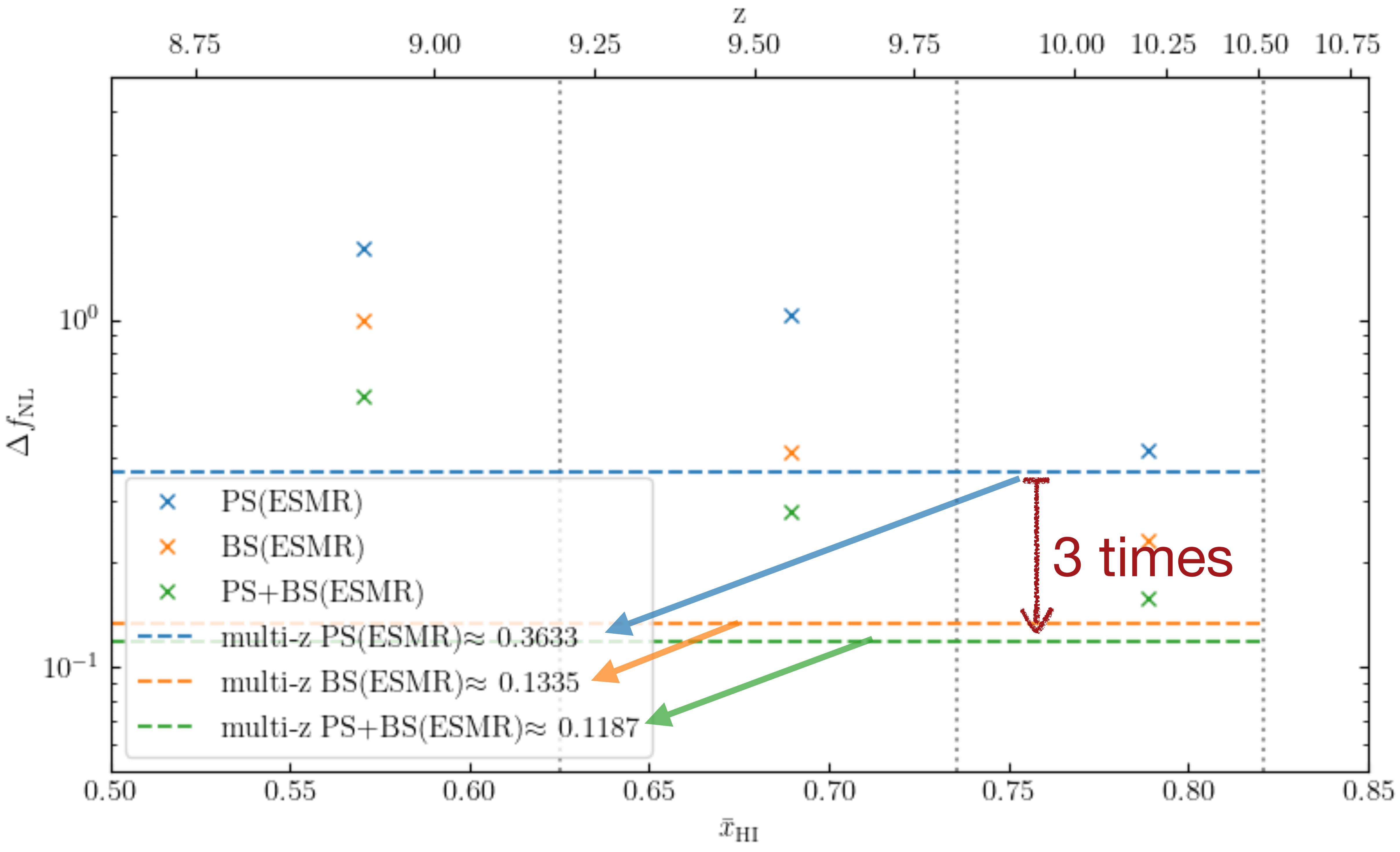

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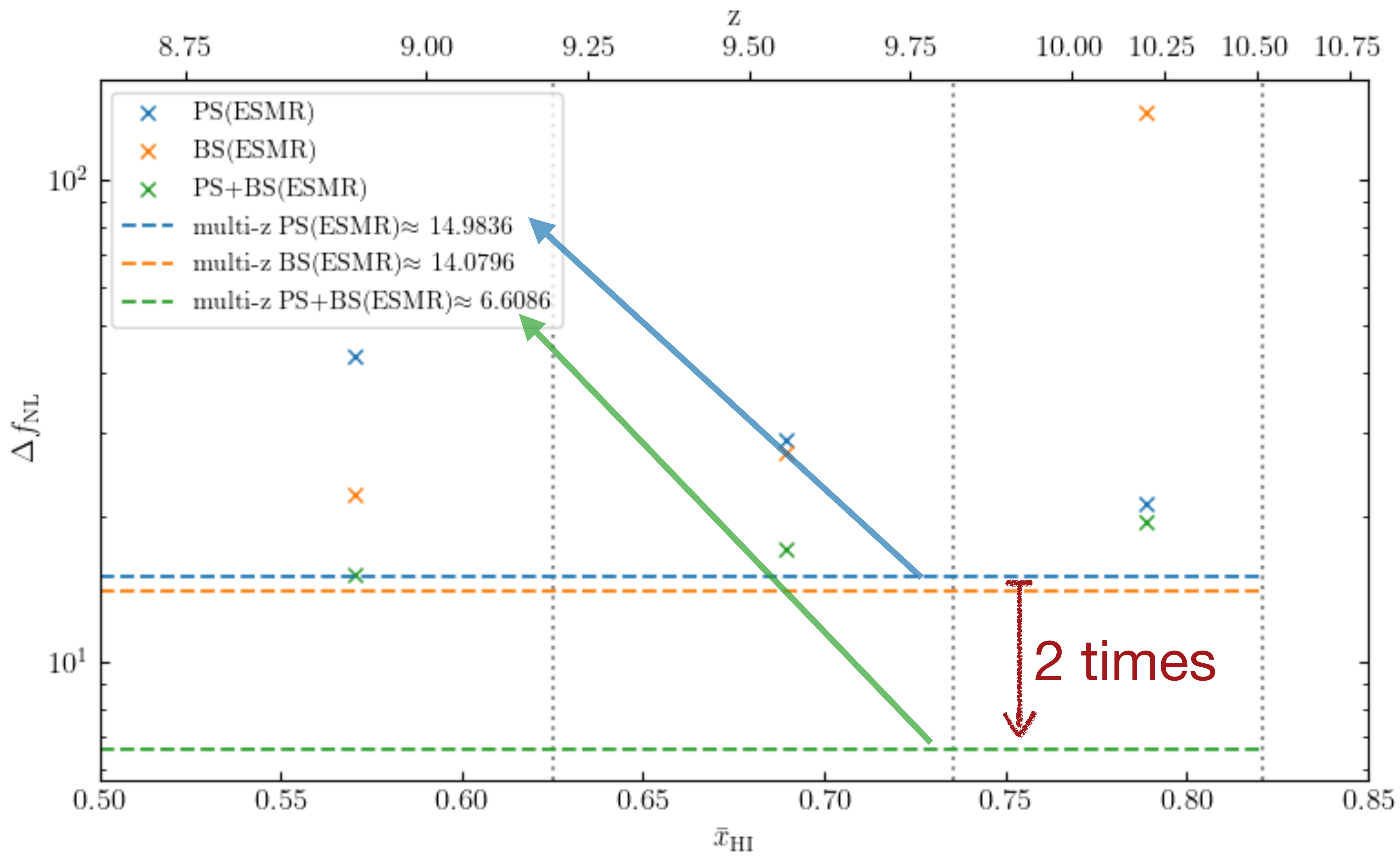

Zhao, Mao & Wang, in prep.

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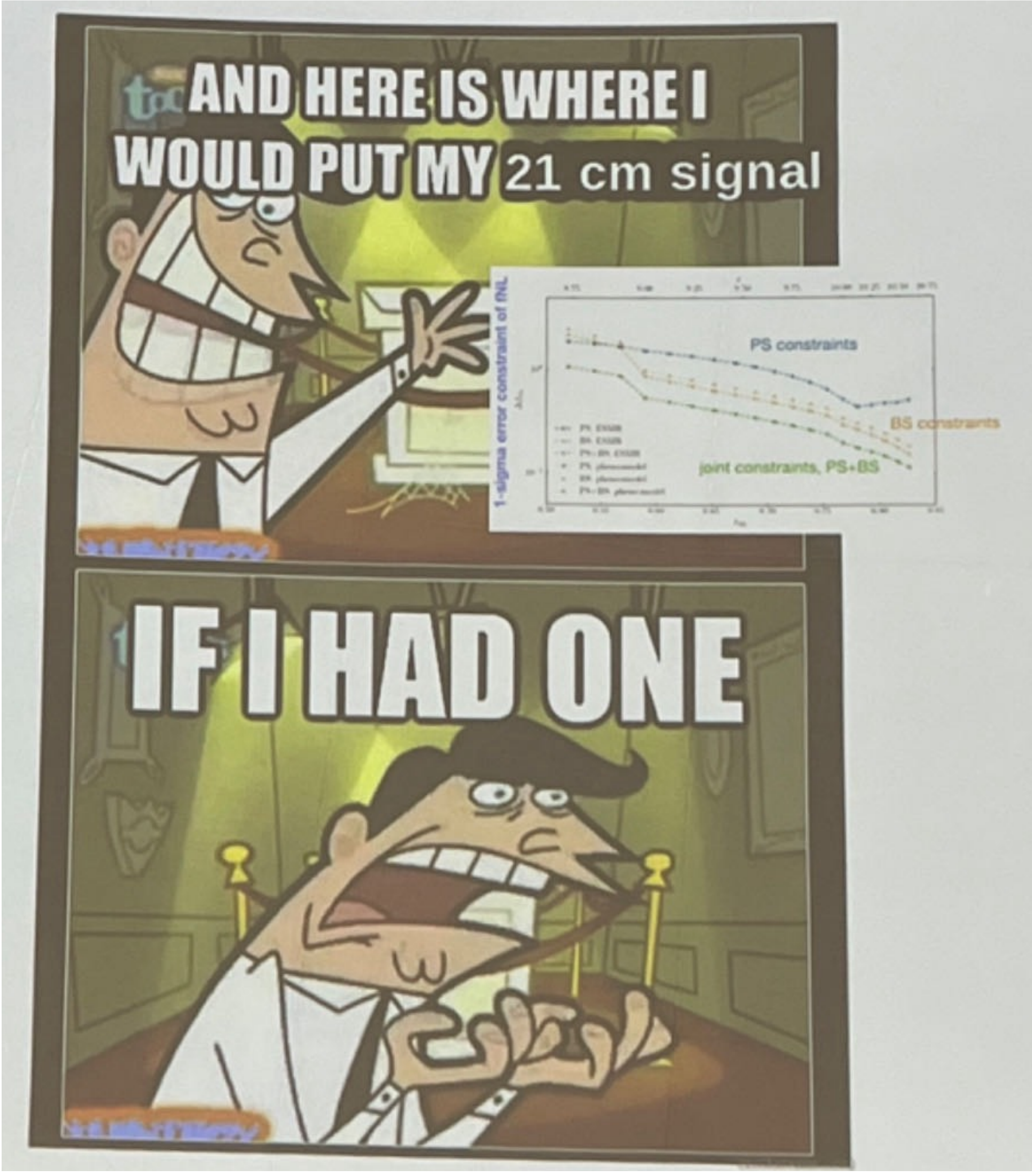


Thank you!

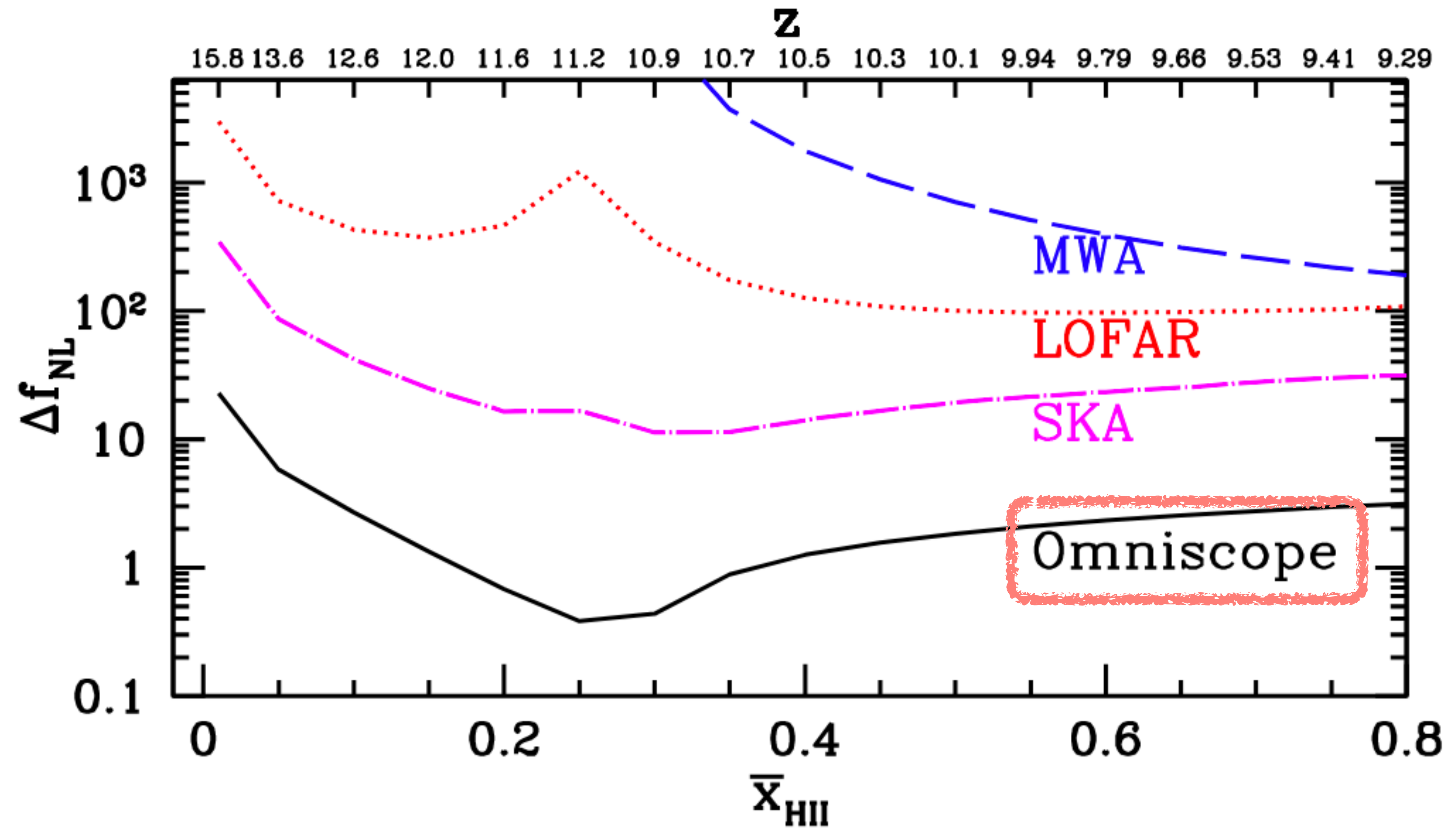
Bake-up follows

Background

Constrain PNG with 21-cm PS from EoR

& BS

$$P_{\Delta T}(k,\mu,z) = \left(\widehat{\delta T_{\rm b}}\,(z_{\rm cos})\right)^2 \left[b_1(z) + \Delta b(k,z) + \mu_{\mathbf{k}}^2\right]^2 P_L(k,z)$$



(Cosmic variance limited)

Mao et al. 2013

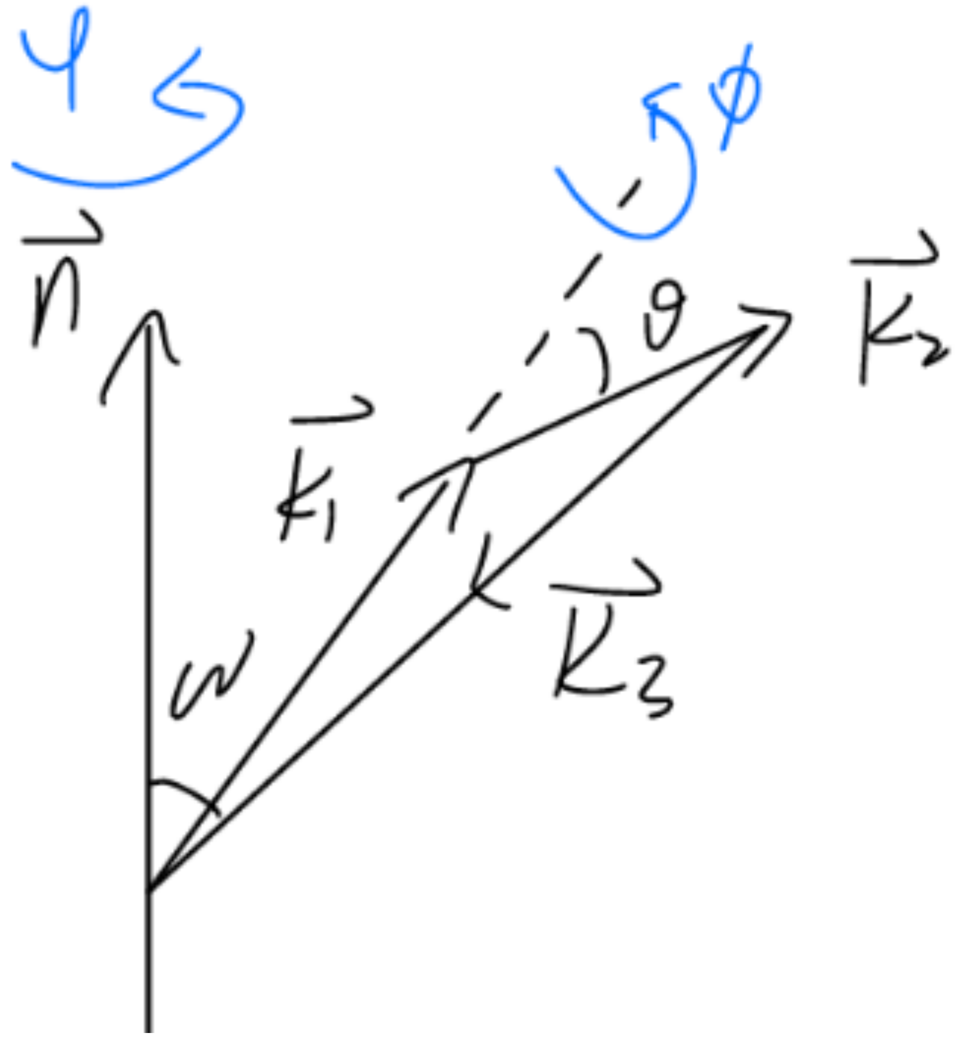
Statistics

- Bispectrum (BS)
- Power Spectrum (PS)

scale-dependent bias

$$\Delta b(k,z) = 2 f_{\rm NL}\delta_{cr}(b_1(z)-1)\mathcal{M}^{-1}(k,z)$$

2 DoF for a k mode
k_perp, k_LOS



5 DoF for a triangle mode
k1_perp, k1_z
k2_x, k2_y, k2_z
-> more samples!

Methodology: Theory

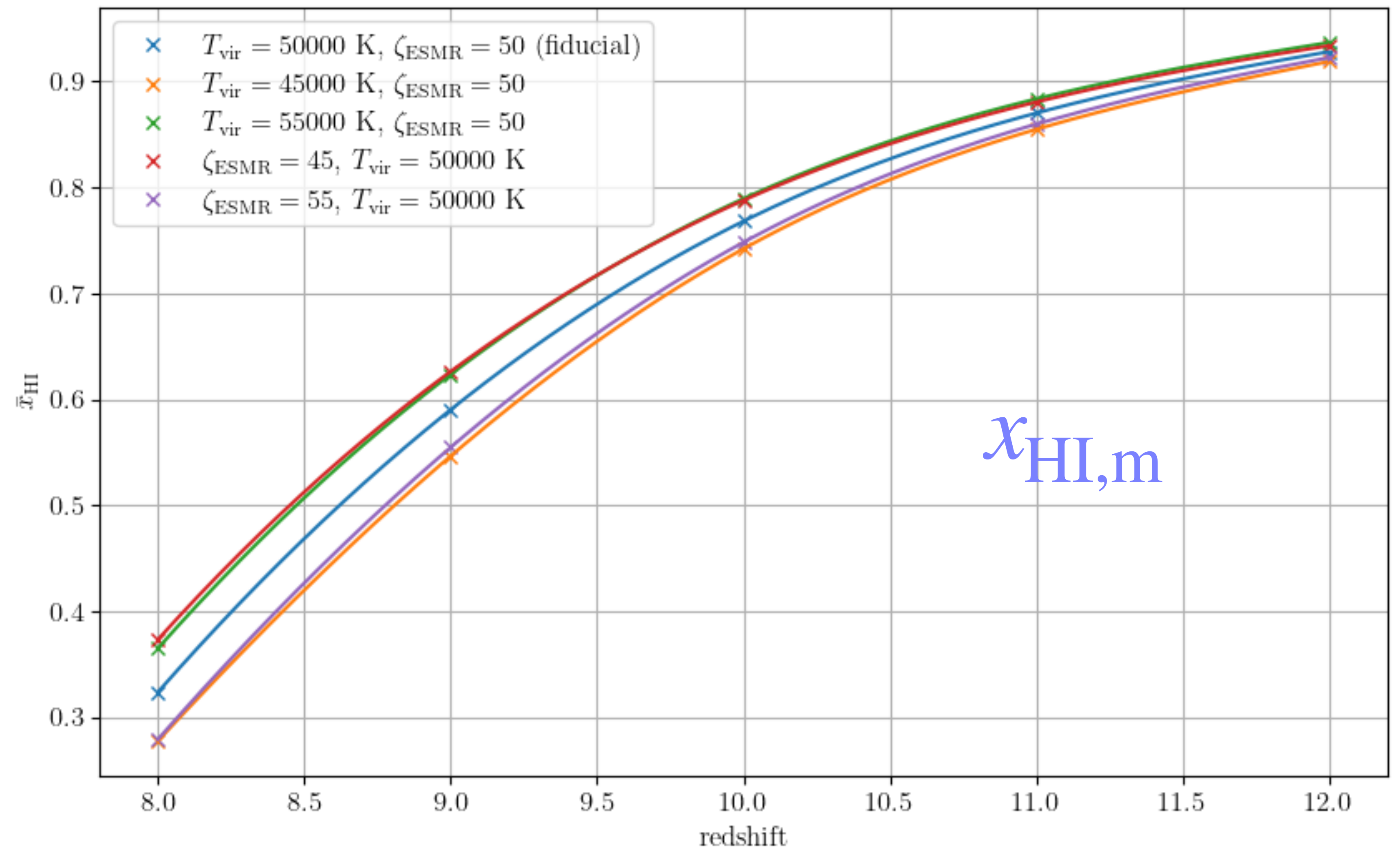
Bias Model

$$\delta_{\rho_{\rm HI}}(\boldsymbol{x}) = b_1 \delta_m(\boldsymbol{x}) + \frac{1}{2} b_2 \delta_m^2(\boldsymbol{x})$$

Fitting EoR history and bias parameters from simulations.

- simulation: 21cmFAST
- box length = 1000Mpc
- low resolution cell number: 512 x 512 x 512
- redshift = [8, 9, 10, 11, 12]
- $T_{\rm vir} = 50000{\rm K} \pm 10\,\%$
- $\zeta = 50 \pm 10\,\%$
- 20 realizations

Zhao, Mao & Wang, in prep.



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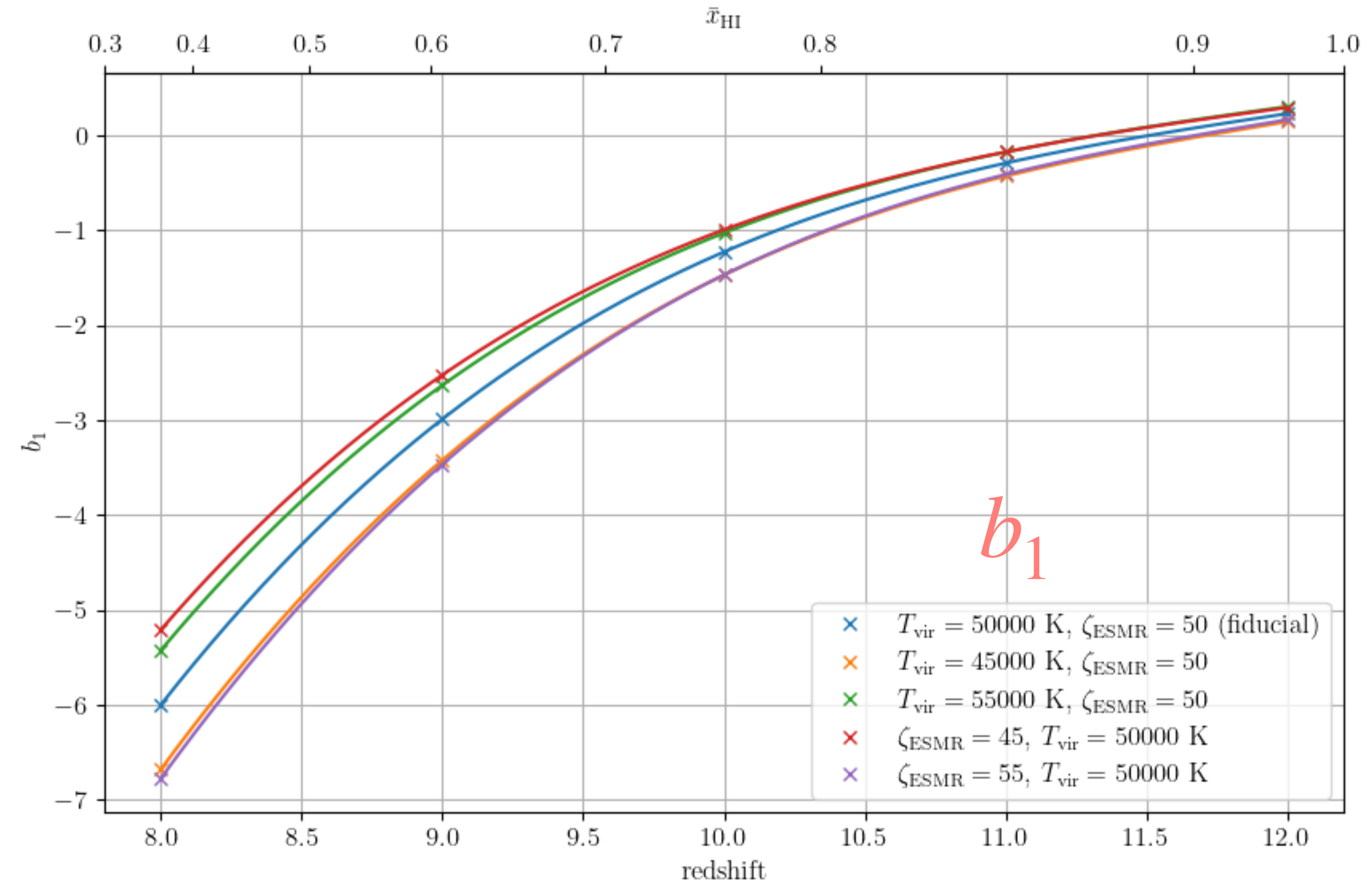

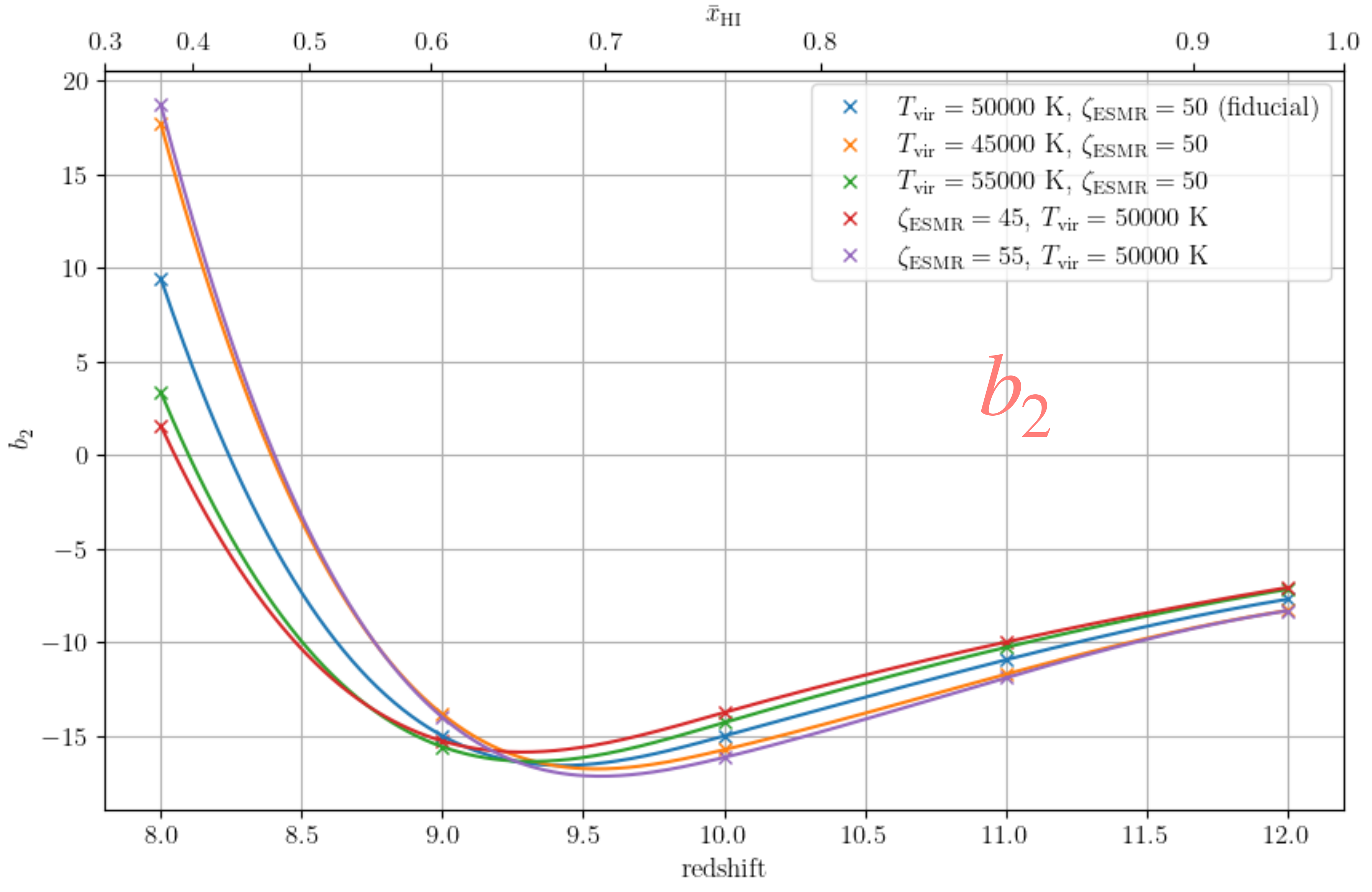

Methodology: Theory

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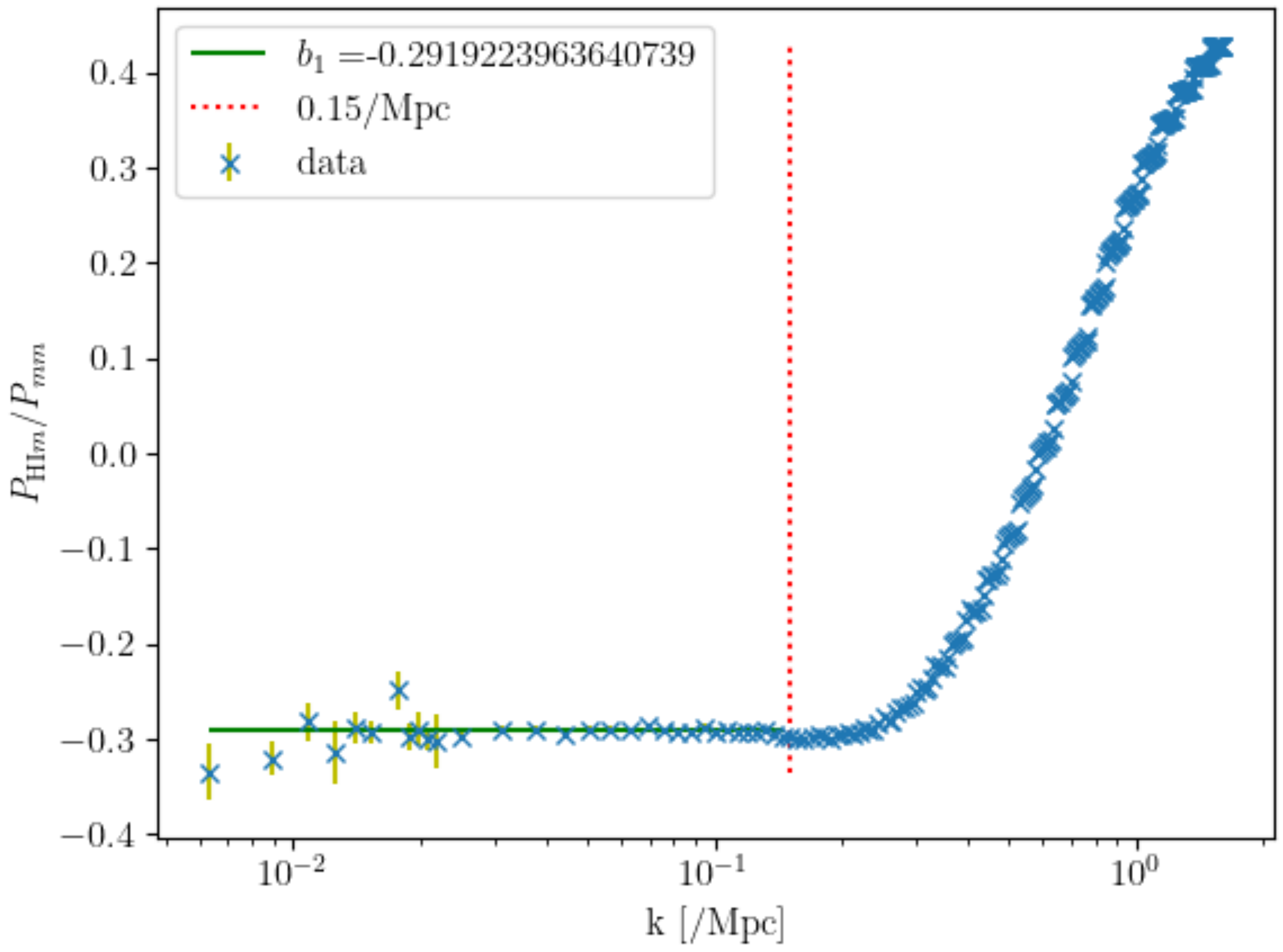

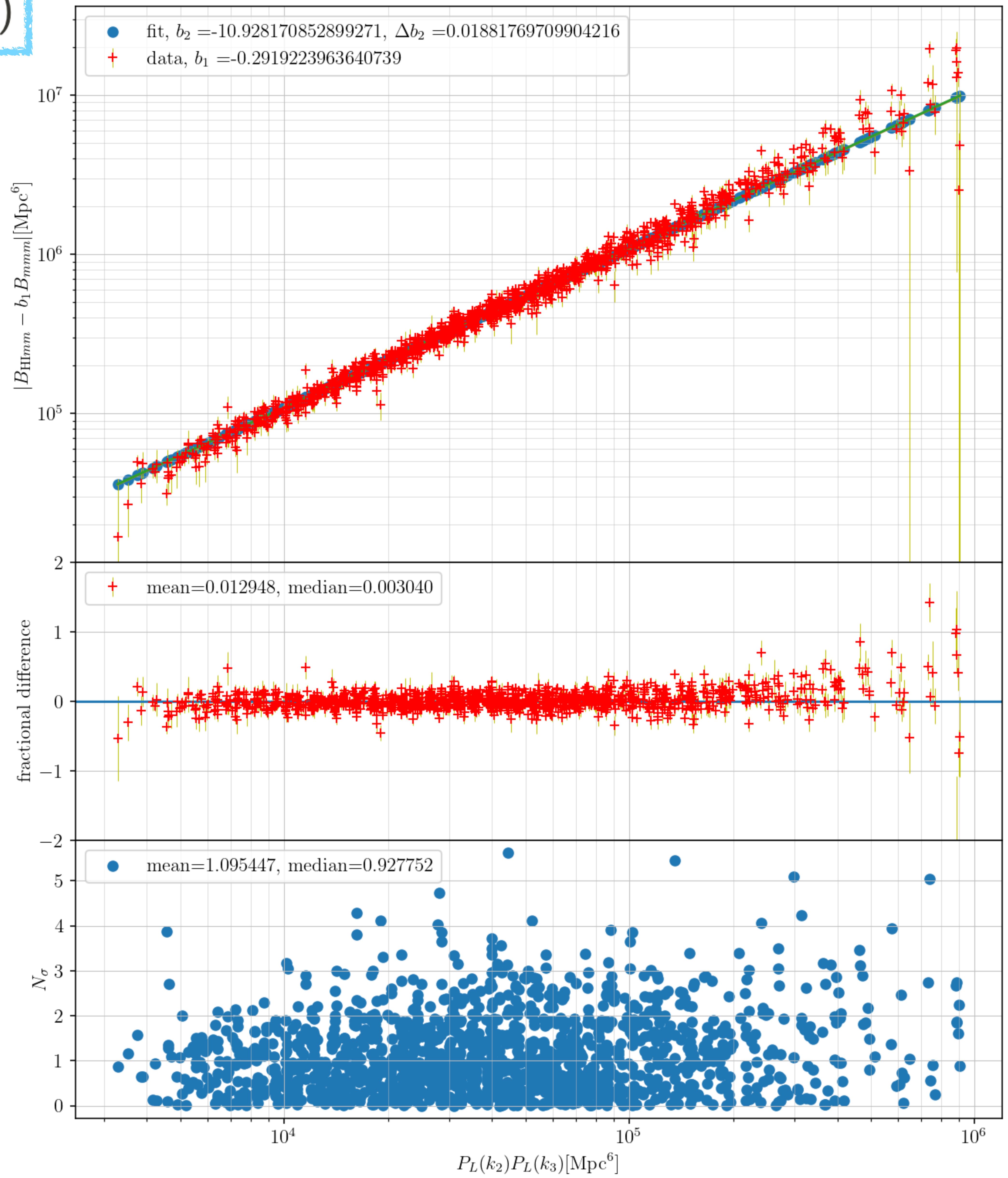

Zhao, Mao & Wang, in prep.

Non-Gaussianity of HI Distribution

What components does HI bispectrum contain?

$$B_{mmm}^{\mathrm{LO}}\left(k_1,k_2,k_3\right)=2F_2\left(\boldsymbol{k}_1,\boldsymbol{k}_2\right)P_{\mathrm{L}}\left(k_1\right)P_{\mathrm{L}}\left(k_2\right)+2\text{perm.}$$

$$F_2\left(\boldsymbol{k}_1,\boldsymbol{k}_2\right)=\frac{5}{7}+\frac{2}{7}\cos^2\theta+\frac{\cos\theta}{2}\left(\frac{k_1}{k_2}+\frac{k_2}{k_1}\right)$$

$$B_{\mathrm{HI,HI,HI}}^{\mathrm{G}} = b_1^3 B_{mmm}^{\mathrm{LO}}+\left[b_1^2 b_2 P_{\mathrm{L}}\left(k_1\right)P_{\mathrm{L}}\left(k_2\right)+2\text{ perm.}\right]$$

$$B_{\mathrm{HI,HI,HI}}=B_{\mathrm{HI,HI,HI}}^{\mathrm{G}}+b_1^3 B_{mmm}^{(1)}+\Big(P_L\left(k_1\right)P_L\left(k_2\right)\Big\{b_1^2\Big[\mathcal{R}_b\left(\Delta b\left(k_1\right)+\Delta b\left(k_2\right)\right)+\mu_{12}\left(\frac{k_1}{k_2}\Delta b\left(k_1\right)+\frac{k_2}{k_1}\Delta b\left(k_2\right)\right)\Big]$$
$$+b_1\left(2F_2\left(\boldsymbol{k}_1,\boldsymbol{k}_2\right)b_1+b_2\right)\left(\Delta b\left(k_1\right)+\Delta b\left(k_2\right)\right)\Big\}+2\text{ perm.}\Big)$$

$$\Delta b(k)=-\frac{\bar{x}_{\mathrm{HII}}}{\bar{x}_{\mathrm{HI}}}2\delta_{\mathrm{cr}}f_{\mathrm{NL}}\left(b_{\rho_{\mathrm{HII}}}^{\mathrm{G}}-1\right)\mathcal{M}^{-1}(k,z)$$

$$\frac{b_{\phi\delta}}{b_\phi}=1+\frac{\delta_{\mathrm{cr}}b_2^L-b_1^L}{2\delta_{\mathrm{cr}}b_1^L}\equiv\mathcal{R}_b$$

$$B_{mmm}^{(1)}=2f_{\mathrm{NL}}\left[P_L\left(k_1\right)P_L\left(k_2\right)\mathcal{M}\left(k_3\right)\mathcal{M}^{-1}\left(k_1\right)\mathcal{M}^{-1}\left(k_2\right)+2\text{ perm.}\right]$$

$$\mathcal{M}(k,z)\equiv\frac{2}{3}\frac{k^2T(k)}{H_0^2\Omega_{\mathrm{m}}}g(0)D(z)$$

$$\delta_{\rho_{\mathrm{HI}}}(\boldsymbol{x})=b_1\delta_m(\boldsymbol{x})+f_{\mathrm{NL}}b_\phi\phi(\boldsymbol{q})+\frac{1}{2}b_2\delta_m^2(\boldsymbol{x})+f_{\mathrm{NL}}b_{\phi\delta}\phi(\boldsymbol{q})\delta_m(\boldsymbol{x})$$

- Nonlinear evolution
  - use Perturbation Theory to deal with the gravitational evolution of matter
- Nonlinear bias (second order)
- Linear growth of PNG
  - assume the PNG of matter field grows linearly.
- PNG effect on bias
  - Potential comes into the bias theory, up to second order.

Bispectrum and Cross-bispectrum in Real Space

Equations

$$b_1^3 B_{mmm}^{\rm LO} + \left[b_1^2 b_2 P_{\rm L}(k_1) P_{\rm L}(k_2) + 2\ \text{perm.}\right]$$

$$\begin{aligned} B_{\rm HI,HI,HI} = &B^{\rm G}_{\rm HI,HI,HI} + b_1^3 B^{(1)}_{mmm} + \Bigg(P_L(k_1) P_L(k_2) \\ &\Big\{ b_1^2 \Big[\mathcal{R}_b \left(\Delta b(k_1) + \Delta b(k_2)\right) + \mu_{12} \left(\frac{k_1}{k_2}\Delta b(k_1) + \frac{k_2}{k_1}\Delta b(k_2) \right) \Big] \\ &+ b_1 \left(2F_2(\boldsymbol{k}_1, \boldsymbol{k}_2) b_1 + b_2\right) \left(\Delta b(k_1) + \Delta b(k_2)\right) \Big\} + 2\ \text{perm.} \Bigg) \end{aligned} \tag{2.28}$$

$$\begin{aligned} B^{\rm r}_{\rm H,HI,HI} = &b_1^2 B^{\rm LO}_{mmm} + b_1^2 B^{(1)}_{mmm} + \Bigg(P_L(k_1) P_L(k_2) \\ &\Big\{ b_1 \Big[b_2 + \mathcal{R}_b \left(\Delta b(k_1) + \Delta b(k_2)\right) + \mu_{12} \left(\frac{k_1}{k_2}\Delta b(k_1) + \frac{k_2}{k_1}\Delta b(k_2) \right) \Big] \\ &+ \left(2F_2(\boldsymbol{k}_1, \boldsymbol{k}_2) b_1 + b_2\right) \Delta b(k_2) \Big\} + (\boldsymbol{k}_2 \leftrightarrow \boldsymbol{k}_3) \Bigg) \\ &+ b_1 \left(\Delta b(k_2) + \Delta b(k_3)\right) 2F_2(\boldsymbol{k}_2, \boldsymbol{k}_3) P_L(k_2) P_L(k_3) \end{aligned} \tag{2.49}$$

$$\begin{aligned} B^{\rm r}_{\rm H,H,HI} = &b_1 B^{\rm LO}_{mmm} + b_2 P_{\rm L}(k_1) P_{\rm L}(k_2) + b_1 B^{(1)}_{mmm} + P_L(k_1) P_L(k_2) \\ &\Big[\mathcal{R}_b \left(\Delta b(k_1) + \Delta b(k_2)\right) + \mu_{12} \left(\frac{k_1}{k_2}\Delta b(k_1) + \frac{k_2}{k_1}\Delta b(k_2) \right) \Big] \\ &+ 2F_2(\boldsymbol{k}_2, \boldsymbol{k}_3) P_L(k_2) P_L(k_3) \Delta b(k_3) \\ &+ 2F_2(\boldsymbol{k}_1, \boldsymbol{k}_3) P_L(k_1) P_L(k_3) \Delta b(k_3) \end{aligned} \tag{2.50}$$

$$B^{\rm LO}_{mmm}(k_1, k_2, k_3) = 2F_2(\boldsymbol{k}_1, \boldsymbol{k}_2) P_{\rm L}(k_1) P_{\rm L}(k_2) + 2\text{perm.}$$

$$F_2(\boldsymbol{k}_1, \boldsymbol{k}_2) = \frac{5}{7} + \frac{2}{7}\cos^2\theta + \frac{\cos\theta}{2}\left(\frac{k_1}{k_2} + \frac{k_2}{k_1}\right)$$

$$B^{(1)}_{mmm} = 2 f_{\rm NL} \mathcal{M}(k_1) \mathcal{M}(k_2) \mathcal{M}(k_3) \left[P_\phi(k_1) P_\phi(k_2) + 2\ \text{perm.}\right]$$

$$\frac{2}{3}\frac{k^2 T(k)}{H_0^2 \Omega_{\rm m}} g(0) D(z) \phi(\boldsymbol{k}) \equiv \mathcal{M}(k,z)\phi(\boldsymbol{k})$$

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$$\frac{b_{\phi\delta}}{b_\phi} = 1 + \frac{\delta_{\rm cr} b_2^L - b_1^L}{2\delta_{\rm cr} b_1^L} \equiv \mathcal{R}_b$$

Methodology: Theory

Redshift Ranges

RSD terms cancellation makes violent redshift evolution.
Need more careful consideration at z>10.5.