NAOC 中国科学院国家天文台 NATIONAL ASTRONOMICAL OBSERVATORIES, CAS

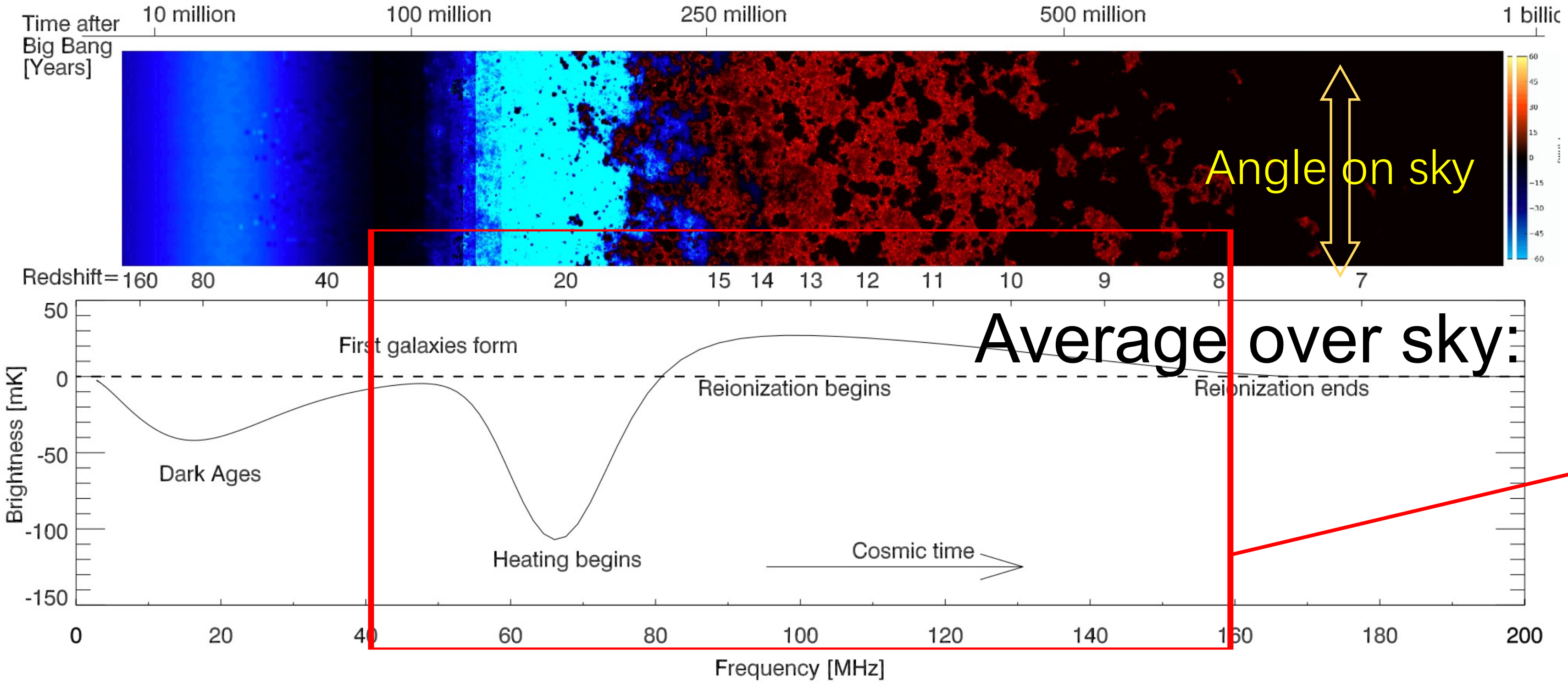
Algorithm of measuring the 21 cm global spectrum with interferometer array

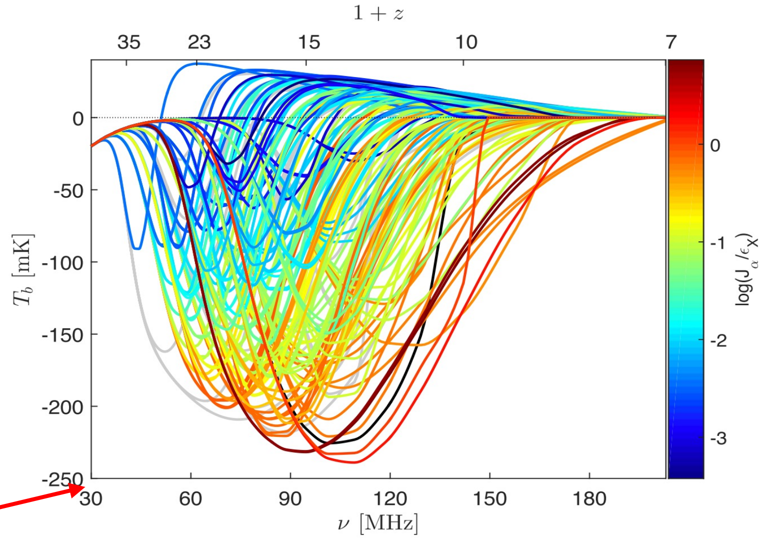
Based on "On measuring the 21 cm global spectrum of cosmic dawn with interferometer array"
Xin Zhang, Bin Yue, Yuan Shi, Fengquan Wu, and Xuelei Chen
2023 ApJ 945 109

Outline

- Introduce to 21cm global spectrum
- Algorithm
- Simulated results
- Summary

21cm Global spectrum





Cohen et al. 2017

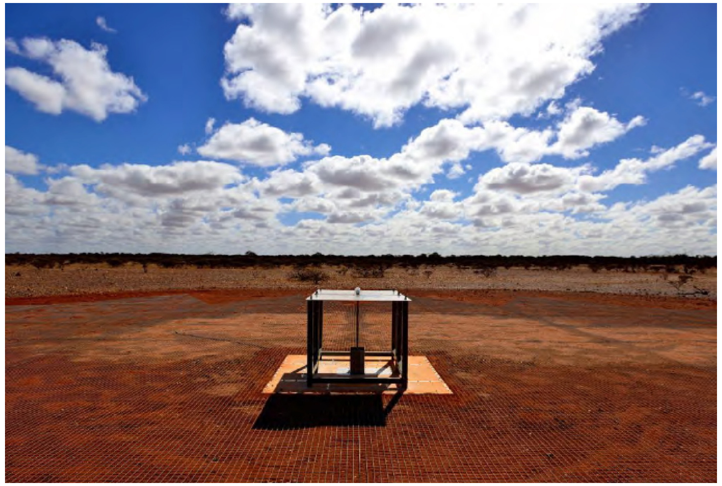
Observable signal : Mean global spectrum $\bar{T}_b(\nu) = \int d\Omega T_b(\hat{\mathbf{r}}, \nu)$

$$\bar{T}_b(z) \sim 27 x_{\rm HI}(z)(1+\delta_b)\left(\frac{H(z)}{dv_r/dr + H(z)}\right)\left(1 - \frac{T_\gamma(z)}{T_{\rm S}(z)}\right)\left(\frac{1+z}{10}\frac{0.15}{\Omega_m h^2}\right)^{1/2}\left(\frac{\Omega_b h^2}{0.023}\right)\mathrm{mK}$$

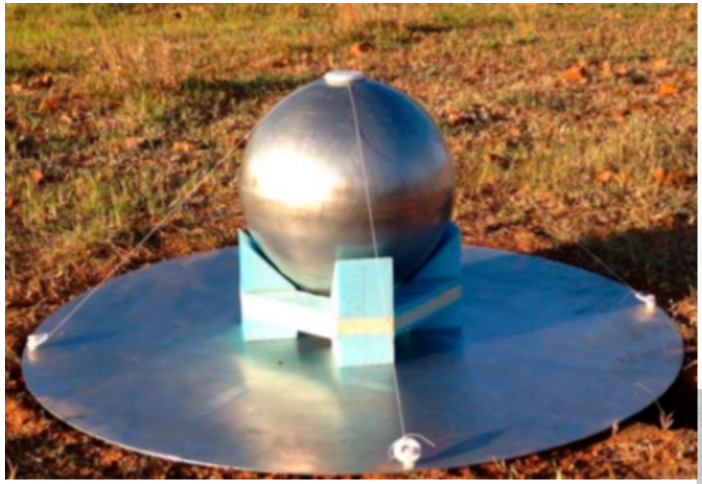
$\bar{T}_b(z) \sim 50\mathrm{mK}$ at Dark age

$\bar{T}_b(z) \sim 200\mathrm{mK}$ at cosmic dawn

How to measure the 21cm Global signal



Experiment to Detect the Global EoR Step
(EDGES)



SARAS2

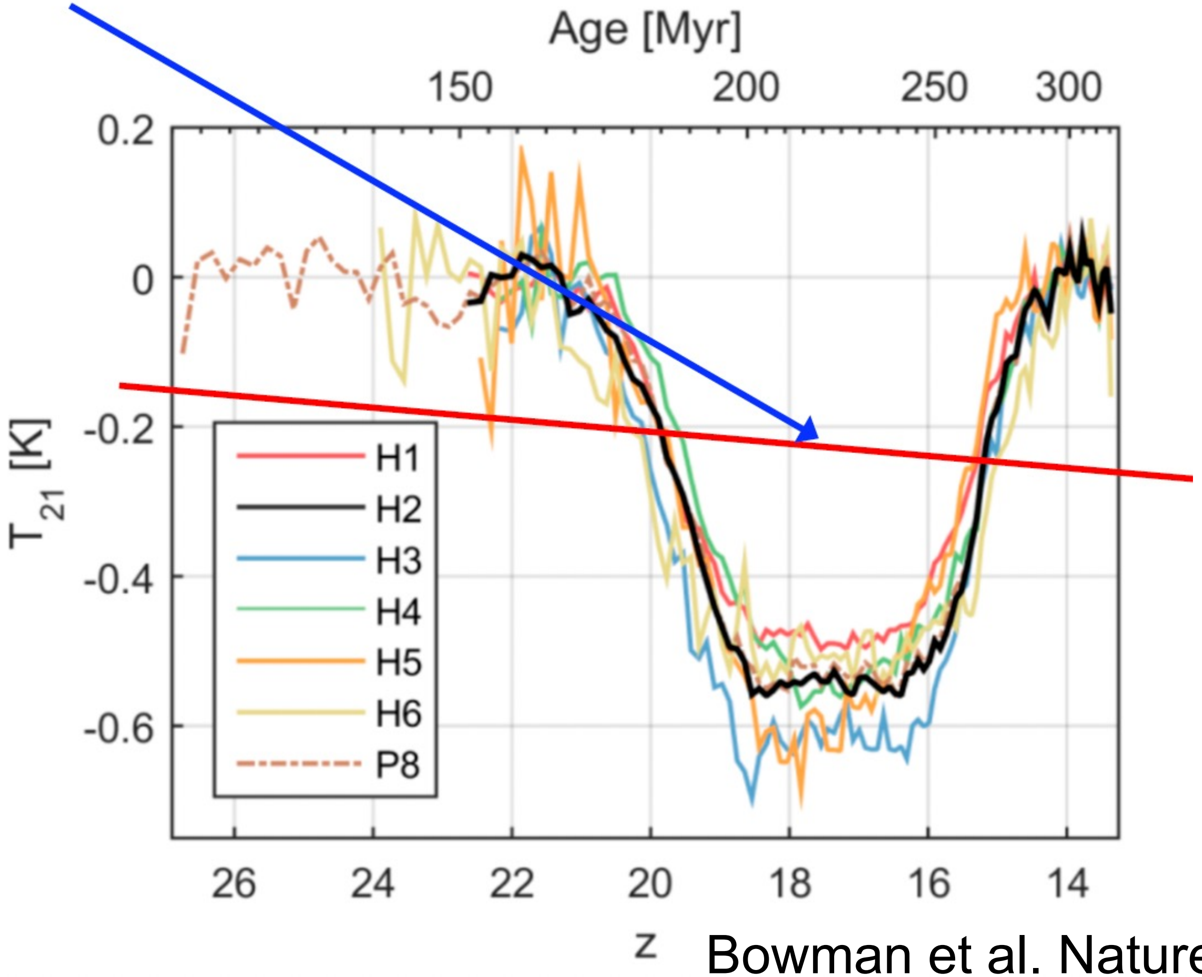
Large aperture Experiment to detect the Dark Ages(LEDA)

Shaped Antenna measurement of the background RAdio Spectrum (SARAS)

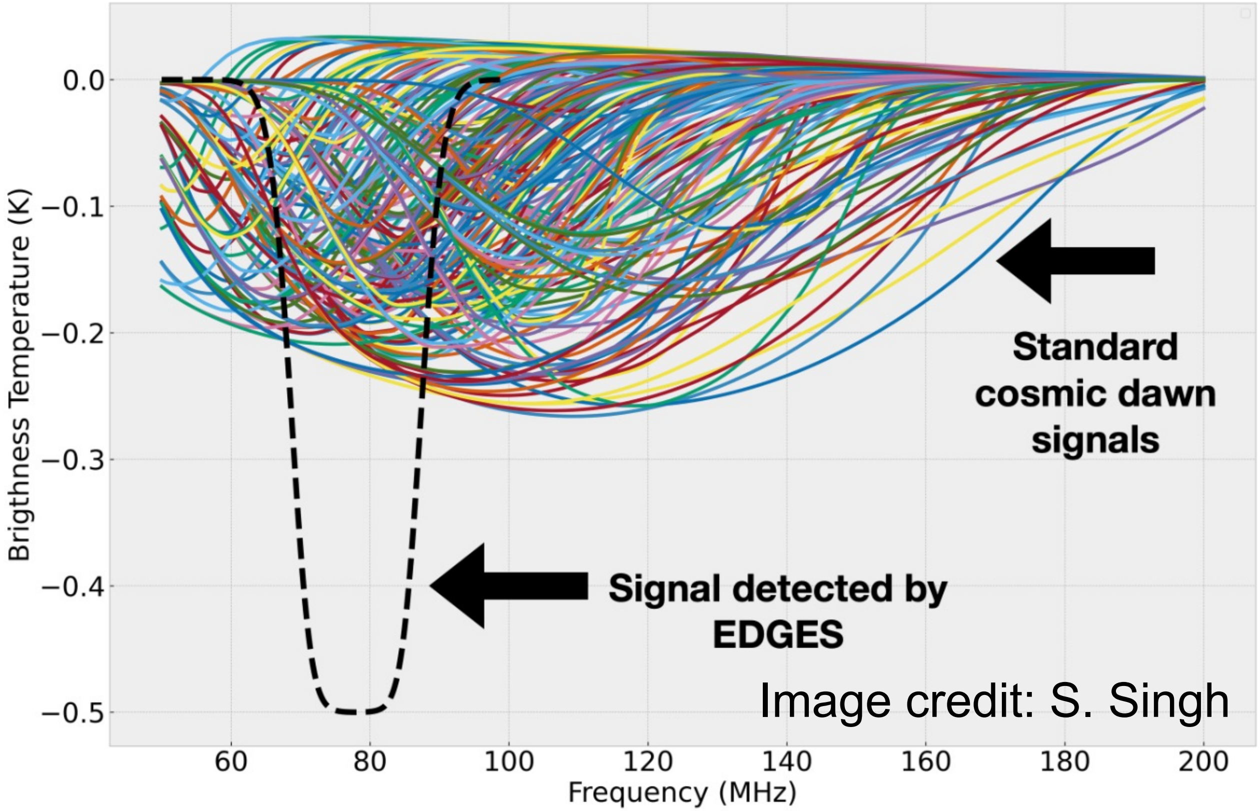
SARAS3

EDGES's result (2018)

Maximum in standard model



EDGES signal is hard to interpret in standard cosmology



Bowman et al. Nature 555,67(2018)

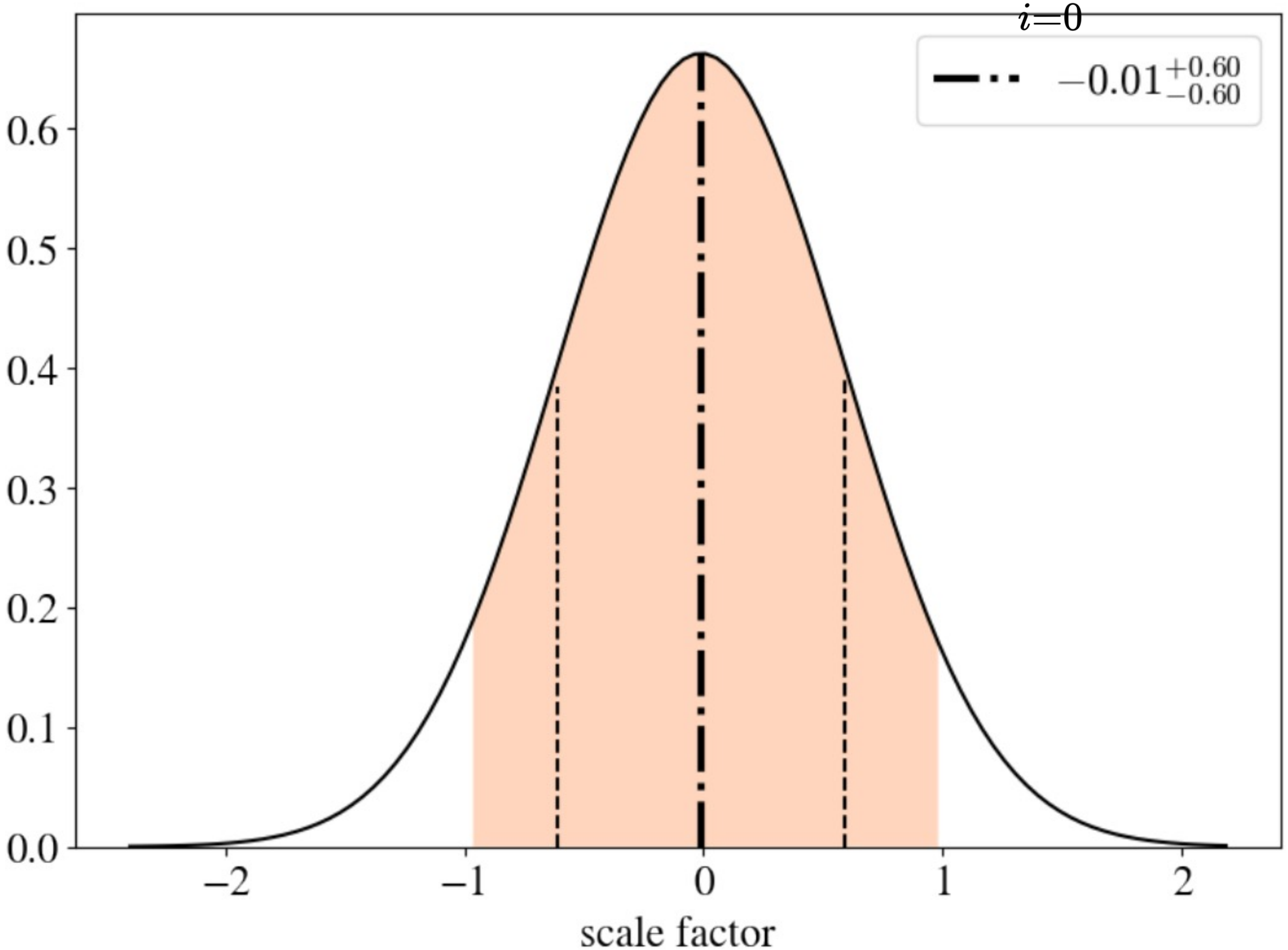
simultaneously. The best-fitting 21-cm model yields a symmetric U-shaped absorption profile that is centred at a frequency of 78±1 MHz and has a full-width at half-maximum of 19^{+4}_{-2} MHz, an amplitude of $0.5^{+0.5}_{-0.2}$ K and a flattening factor of $\tau = 7^{+5}_{-3}$ (where the

SARAS3's result in 2022

Fit the SARAS3 spectrum with a model representing the foreground plus calibration systematics along with the EDGES profile.

$$\log_{10}\{(T(\nu)/1\ \mathrm{K}) - s \times (T_{\mathrm{EDGES}}(\nu)/1\ \mathrm{K})\} = \sum_{i=0}^{i=0} a_i \mathfrak{R}(\log_{10}(\nu/1\mathrm{MHz}))^i$$

S.Singh et al.
Nature Astron. 6 (2022) 5, 607-617



SARAS3

Alternatively, detect the 21cm global spectrum by an interferometer:

It measures the cross-correlation instead of auto-correlation, has different systematics compared to single antenna

An interferometer array has higher angular resolution, therefore it can use celestial objects as calibration sources.

Algorithm and its feasibility

The visibility measured by the interferometer

$$V_\nu(\boldsymbol{b}, \hat{\boldsymbol{n}}_0) = \int d\Omega(\hat{\boldsymbol{n}}) B_\nu(\hat{\boldsymbol{n}}, \hat{\boldsymbol{n}}_0) T_\nu(\hat{\boldsymbol{n}}) e^{-2\pi i \frac{b}{\lambda} \cdot \hat{\boldsymbol{n}}}$$

The spherical harmonic expansion of the sky temperature:

$$T_\nu(\hat{\boldsymbol{n}}) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_l^m Y_l^m(\hat{\boldsymbol{n}})$$

Allow the visibility expanded up to lmax:

$$V_v(\boldsymbol{b}, \hat{\boldsymbol{n}}_0) \approx \sum_{l=0}^{l_{\max}} \sum_{m=-l}^{l} a_l^m \left(\int d\Omega(\hat{\boldsymbol{n}}) B_v(\hat{\boldsymbol{n}}, \hat{\boldsymbol{n}}_0) Y_l^m(\hat{\boldsymbol{n}}) e^{-2\pi i \frac{b}{\lambda} \cdot \hat{\boldsymbol{n}}} \right)$$

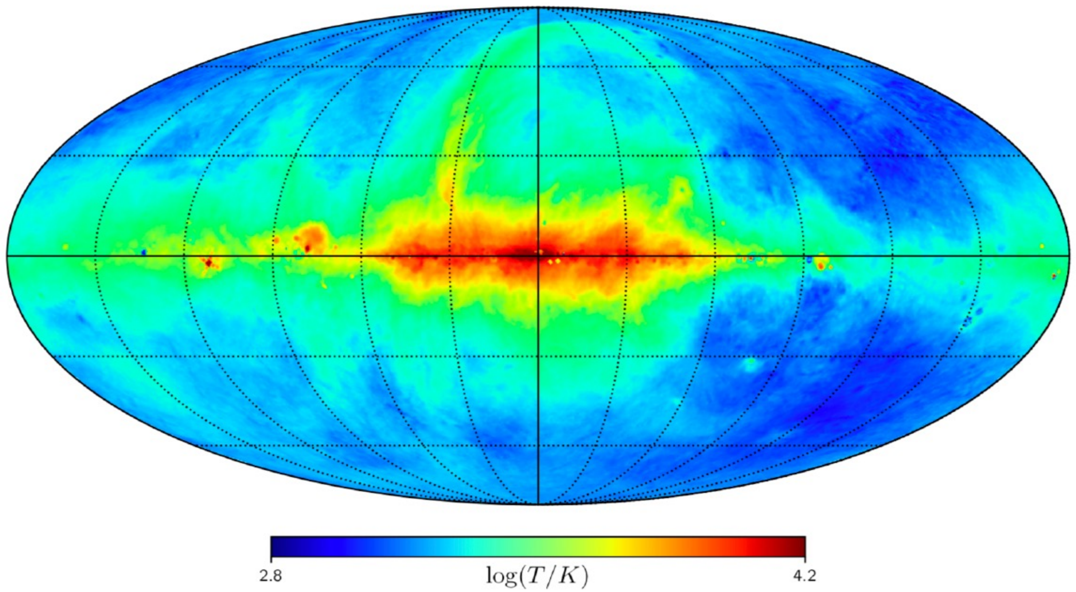
For many baselines it forms matrix form: $\boldsymbol{V} = \boldsymbol{Q}\boldsymbol{a}$

The estimator: $\hat{\boldsymbol{a}} = \boldsymbol{Q}^{-1}\boldsymbol{V}$ finds the minimum-norm least-squares solution

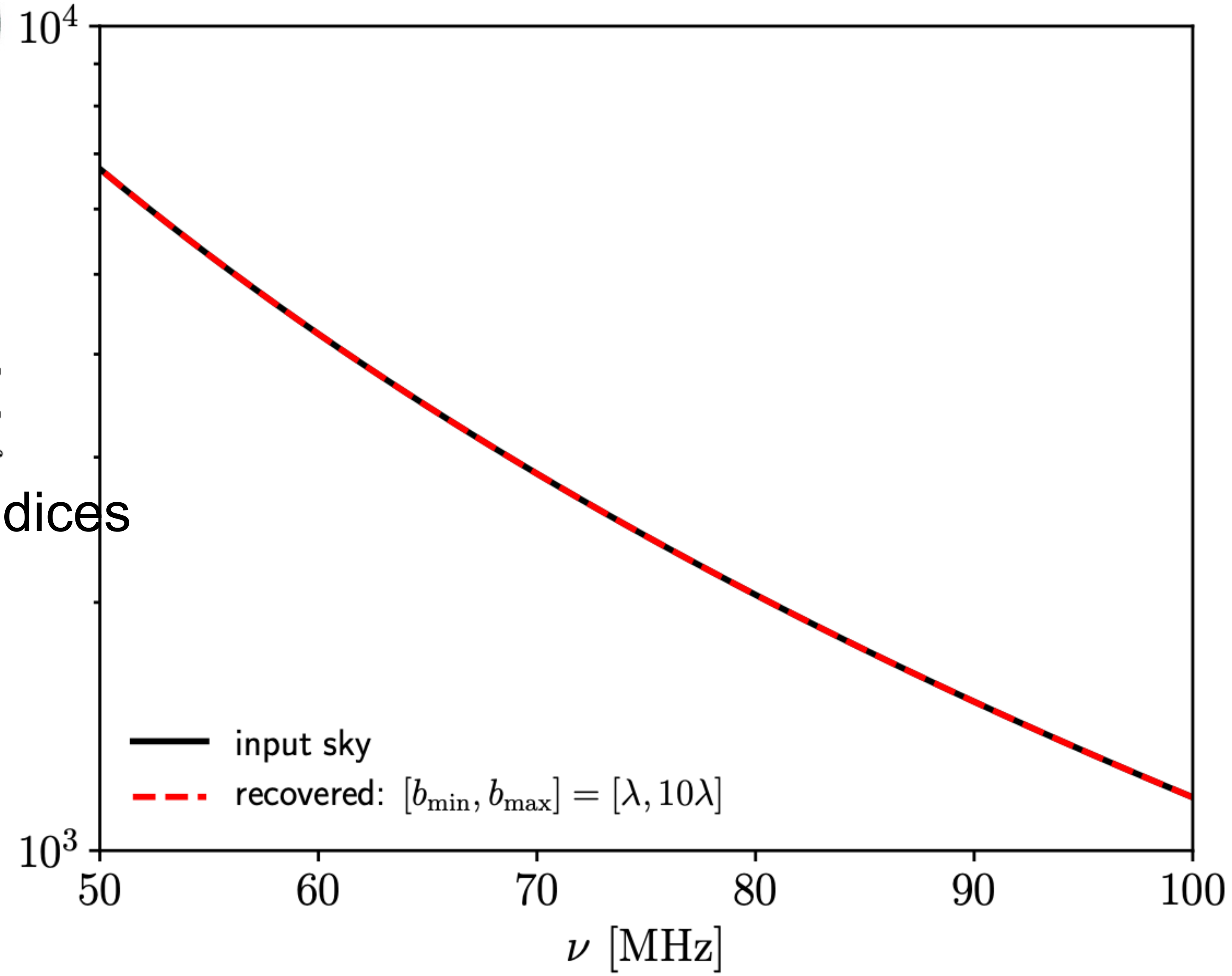
The recovered global sky temperature $\hat{T_0} = \hat{a}_0^0/\sqrt{4\pi}$

For isotropic beam $B_\nu(\hat{\boldsymbol{n}}, \hat{\boldsymbol{n}}_0) = 1$
and randomly 2D baseline:

$$N_L = 4000 \qquad \lambda < b < 10\lambda$$



Input Map:
the ULSA sky model
with direction dependent spectrum indices
(Cong et al. 2021)
See the next talk!



Add a complex Gaussian random noise:

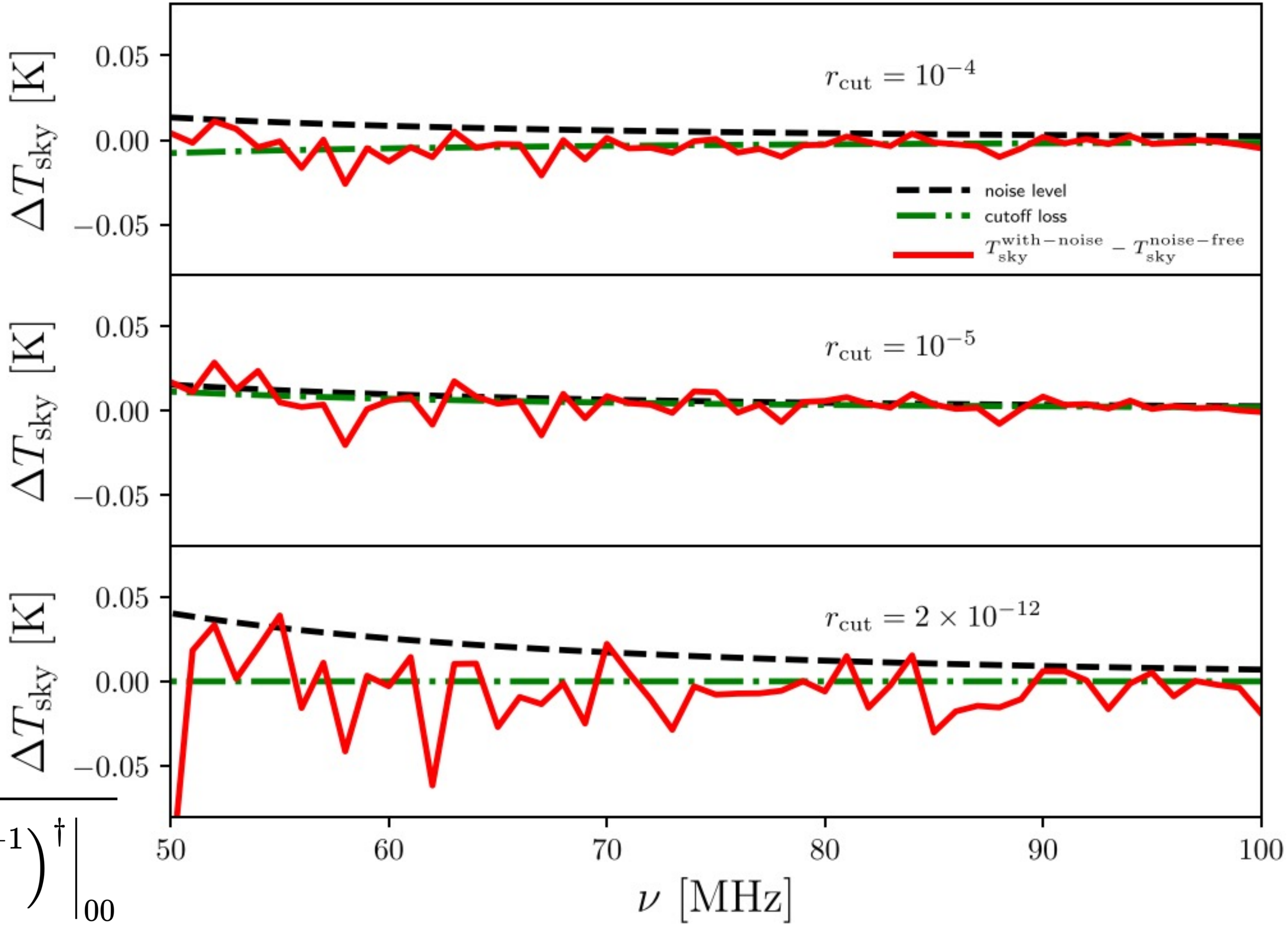
$$\sigma_V(v) = \Omega_B \frac{T_{\mathrm{sky}}(v)}{\sqrt{2\Delta v t_{\mathrm{obs}}}}$$

⇩

$$\boldsymbol{V} = \boldsymbol{Q}\boldsymbol{a} + \boldsymbol{V}_{\mathrm{N}}$$

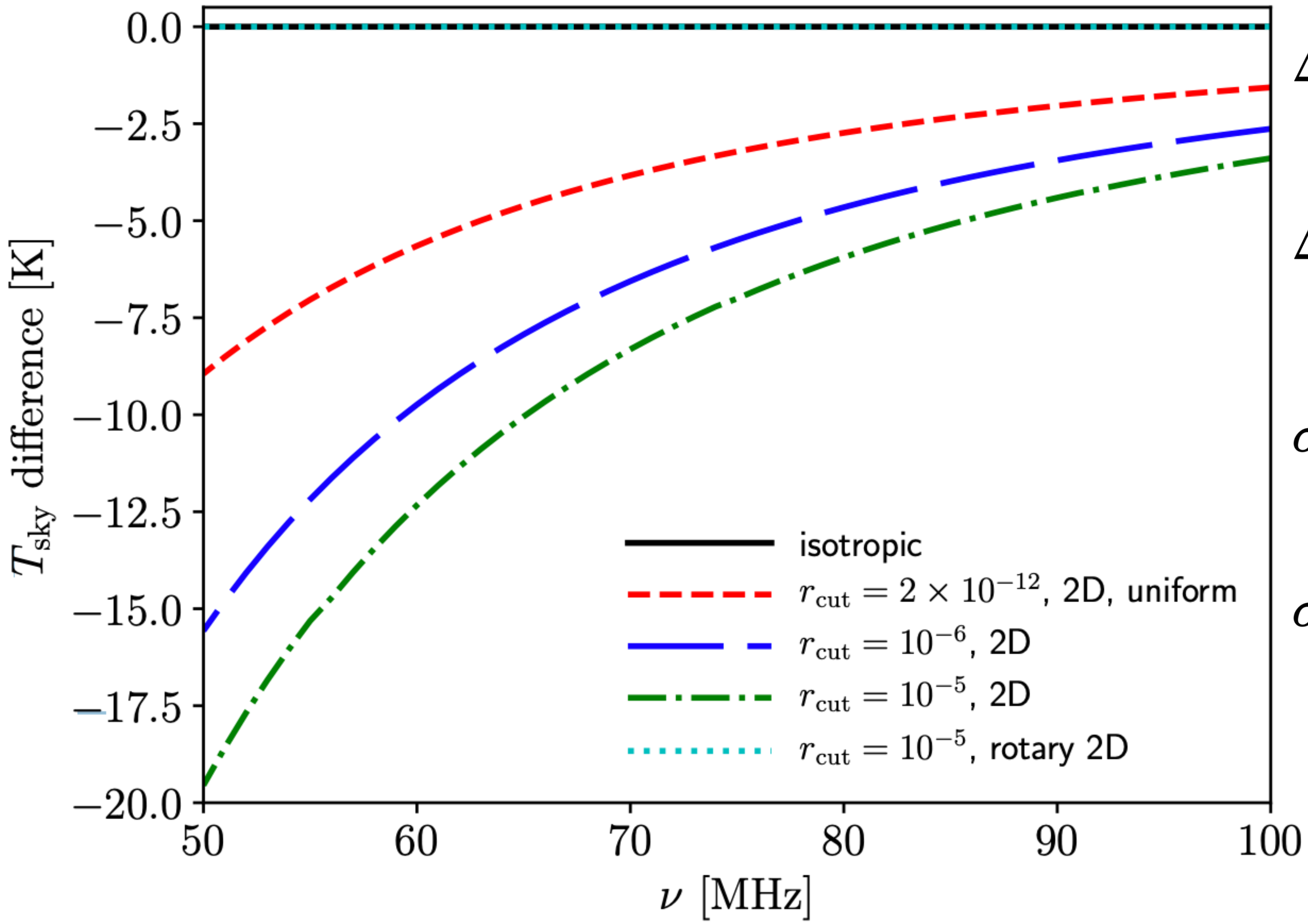
The final noise level on the recovered global spectrum is

$$\sigma_{\mathrm{N}} \sim \frac{1}{\sqrt{4\pi}} \sqrt{\left| \tilde{\boldsymbol{Q}}^{-1} \boldsymbol{N}_V \left(\tilde{\boldsymbol{Q}}^{-1} \right)^{\dagger} \right|_{00}}$$



Dipole beam: $B_\nu(\theta') \propto \frac{\left[\cos\left(\frac{\pi L}{\lambda}\cos\theta'\right) - \cos\left(\frac{\pi L}{\lambda}\right)\right]^2}{\sin^2\theta'}$

$N_L = 4000 \quad \lambda < b < 10\lambda$



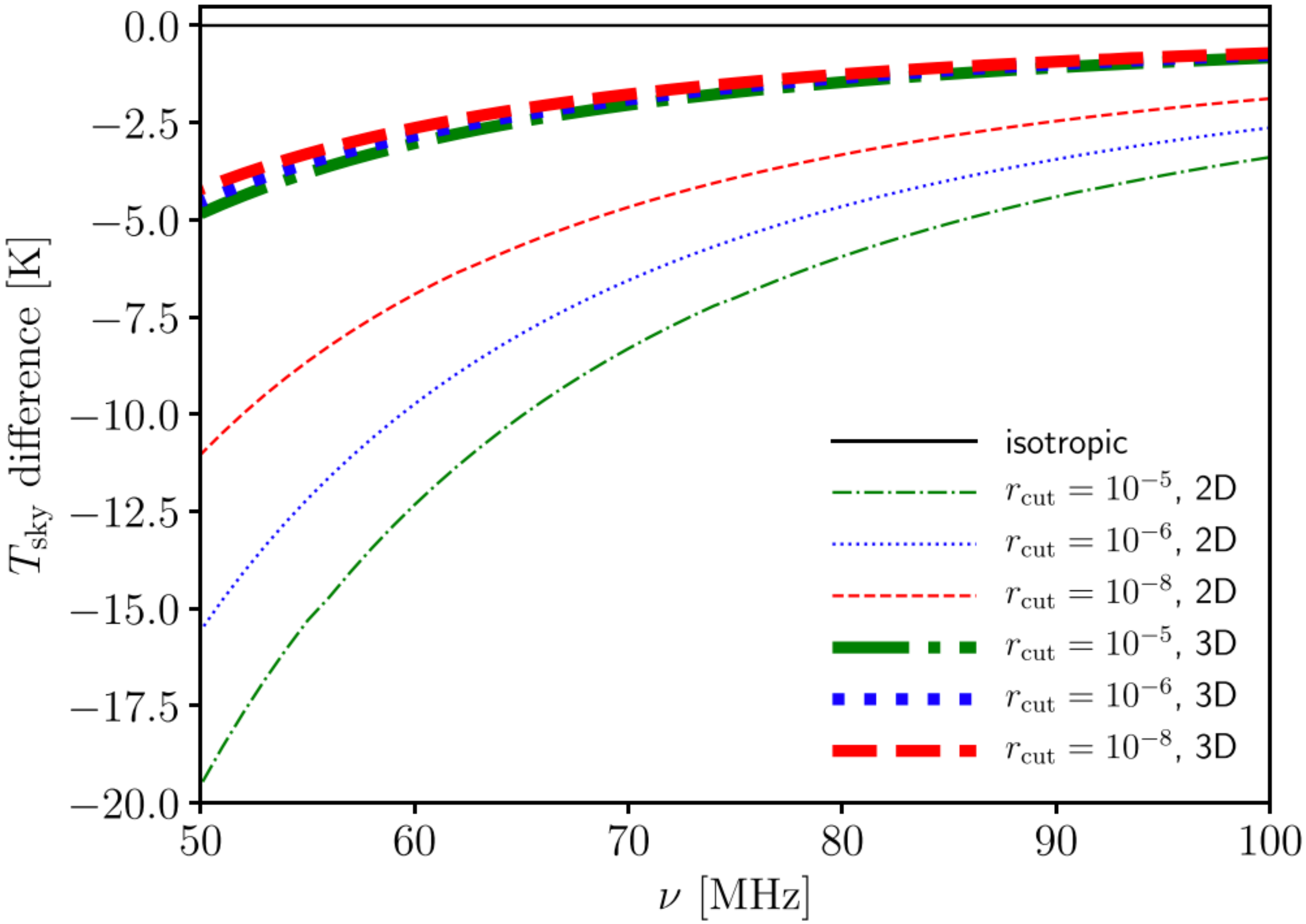
$\Delta T_0 = \hat{T}_0 - T_0$ is $\mathcal{O}(0.1\%)$ of T_0

ΔT_0 depends on $\boldsymbol{b}$ distribution and $B_\nu(\theta')$

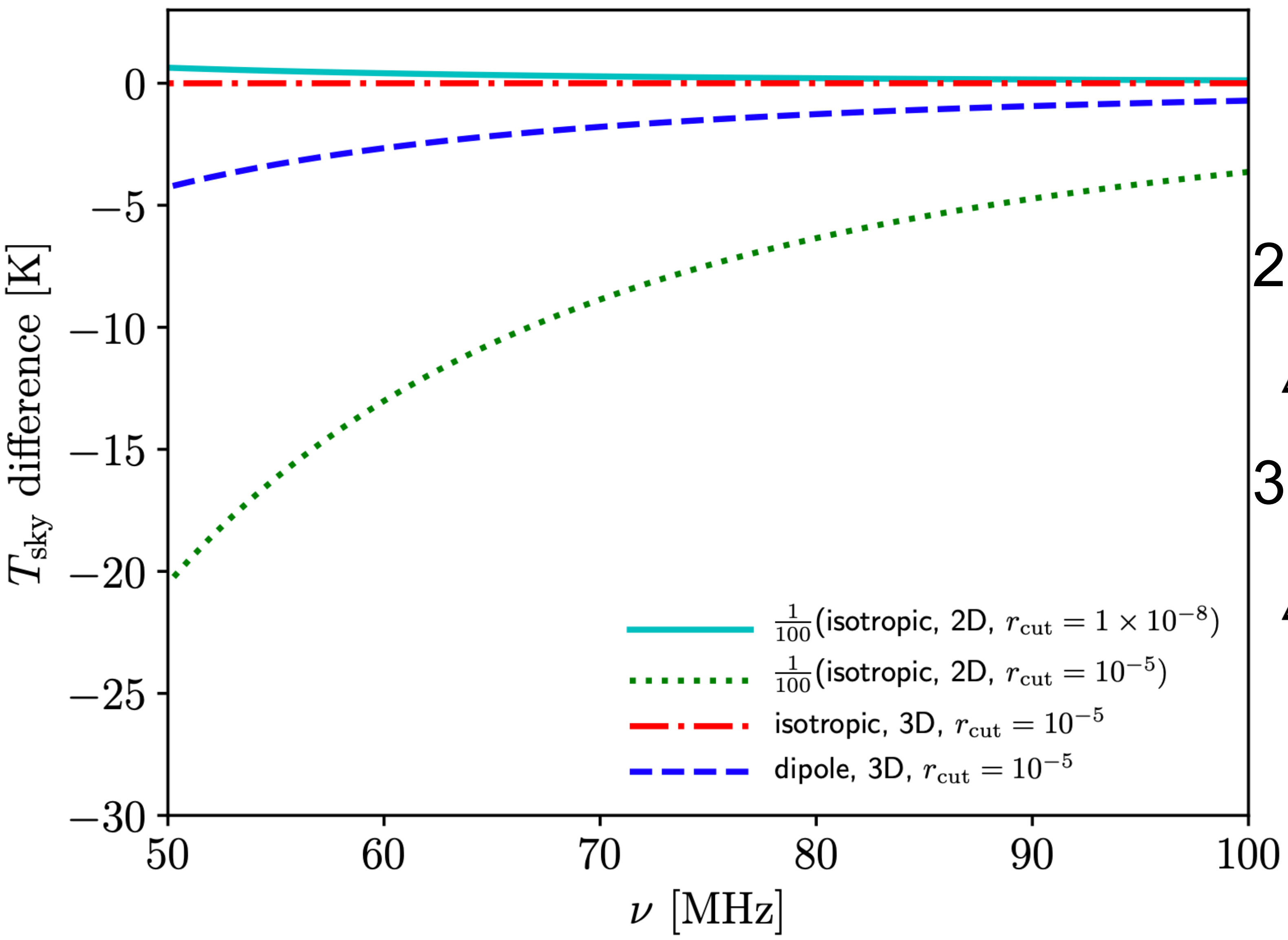
$\sigma(\Delta T_0)$ is $\mathcal{O}(0.01\%)$ of T_0

$\sigma(\Delta T_0) \downarrow$ when $N_L \uparrow$

3D Baselines



The Shortest Baselines



2D baselines with $3\lambda < b < 10\lambda$

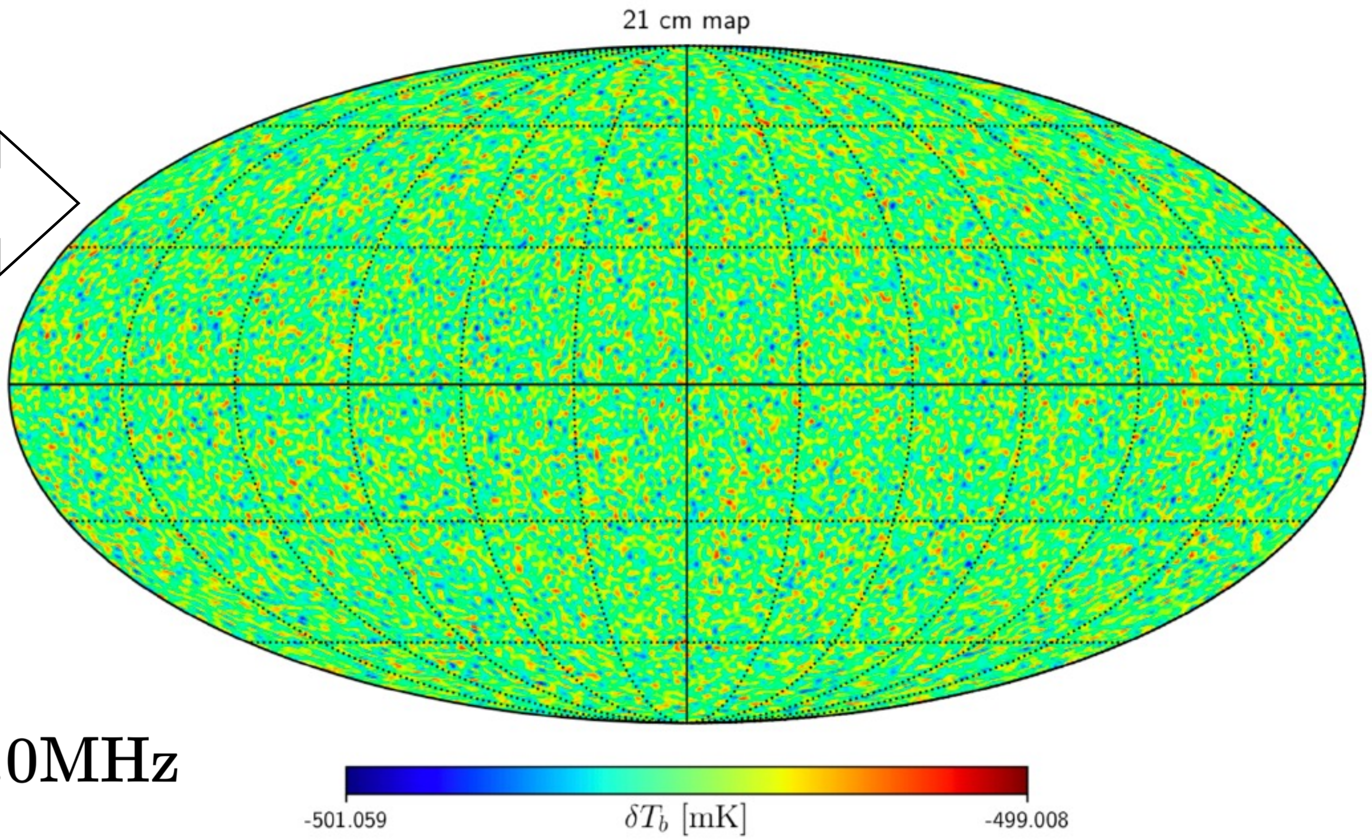
$\Delta T_0 \sim \mathcal{O}(10\%)$

3D baselines with $3\lambda < b < 10\lambda$

$\Delta T_0 \sim \mathcal{O}(0.1\%)$

Extract the 21 cm signal

$$C_l(z) = \frac{2}{\pi}\int dk k^2 P_{21}(k,z)\mathcal{J}_l^2(kr(z))$$



21 cm global signal:

$$\delta T_{21}(\nu) = A\exp\left[-\frac{(\nu-\nu_{21})^2}{2\sigma_{21}^2}\right]$$

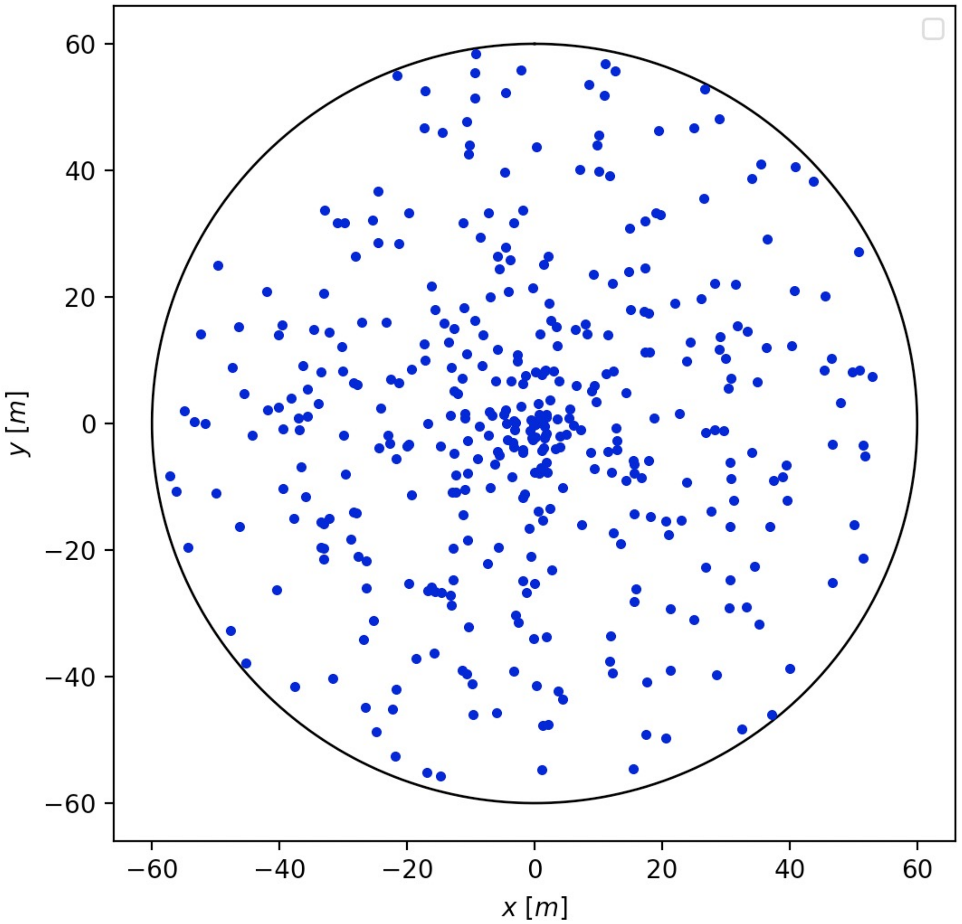
$$A = -0.50\ \mathrm{K}, \nu_{21} = 75.0\mathrm{MHz}, \sigma_{21} = 5.0\mathrm{MHz}$$

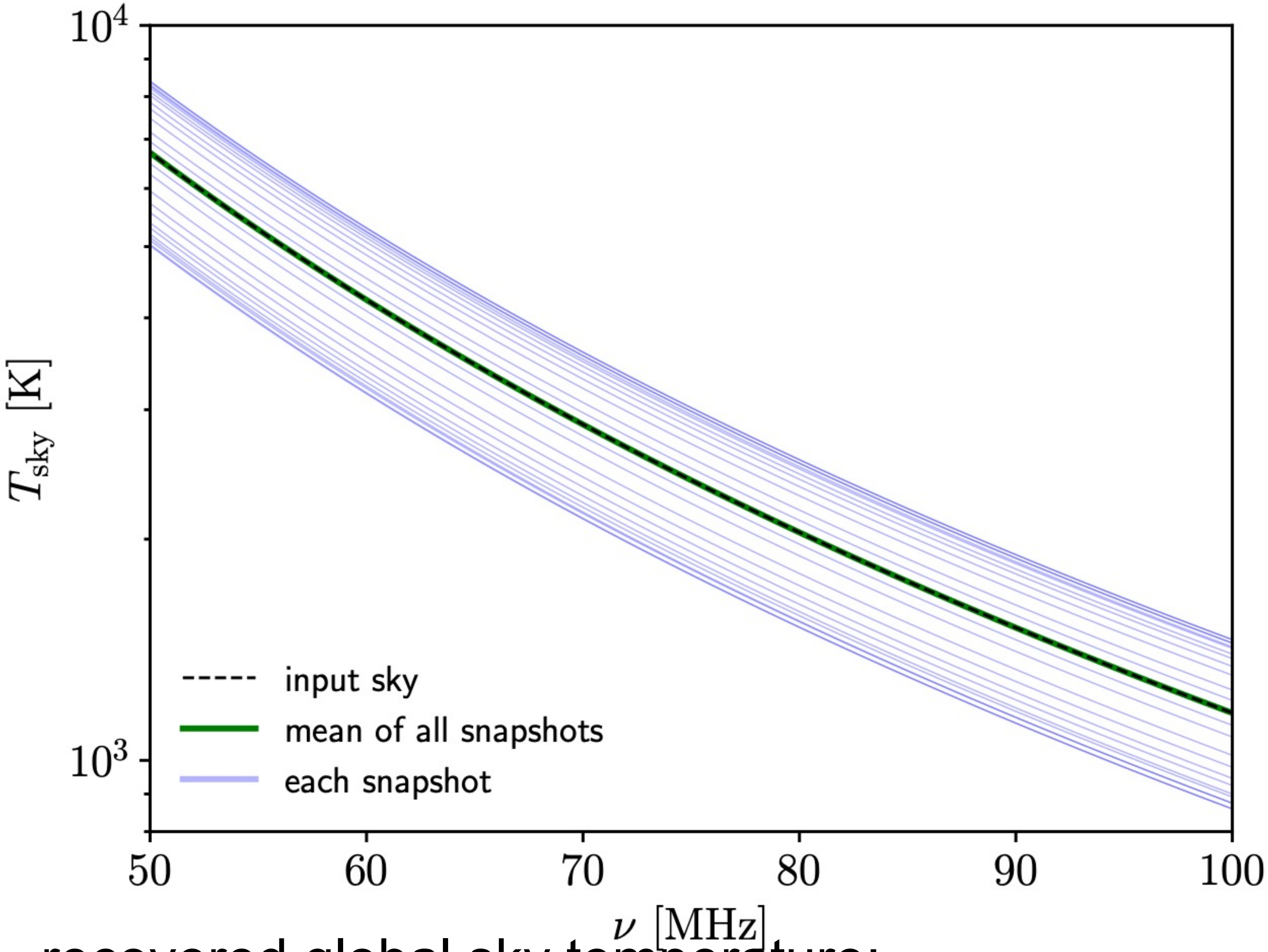
Foreground: $T_{\rm FG}(\nu) = \left(\frac{\nu}{\nu_0}\right)^{-2.5}\left[T_0 + \sum_{i=1}^{4} a_i\left(\log\frac{\nu}{\nu_0}\right)^i\right]$ Shi et al. (2022a).

Markov Chain Monte Carlo analysis: $\chi^2 = \sum_i \frac{\left[T_{\rm FG}(\nu_i) + T_{21}(\nu_i) - \hat{T}_{\rm sky}(\nu_i)\right]^2}{\sigma_{\nu_i}^2}$

Simulated results: ground-based 2D array:

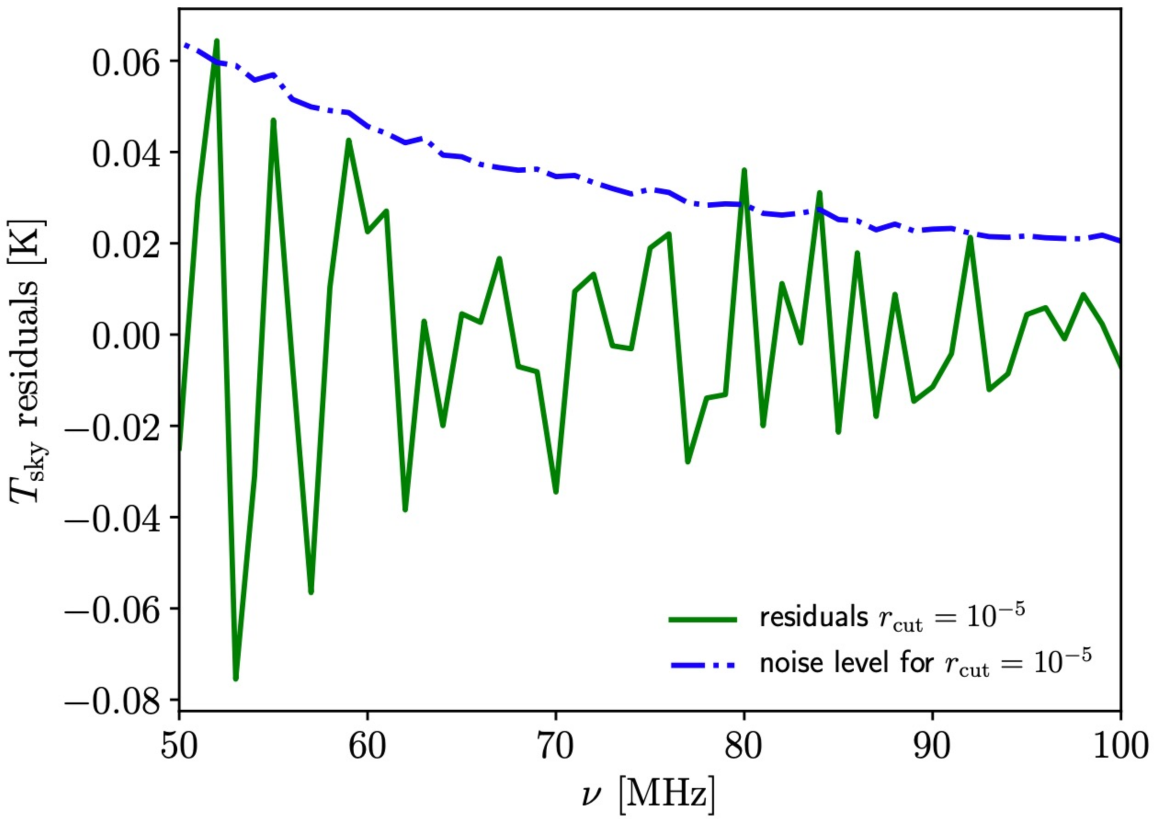
array configuration :
400 antennas; 60m radius,
random locations

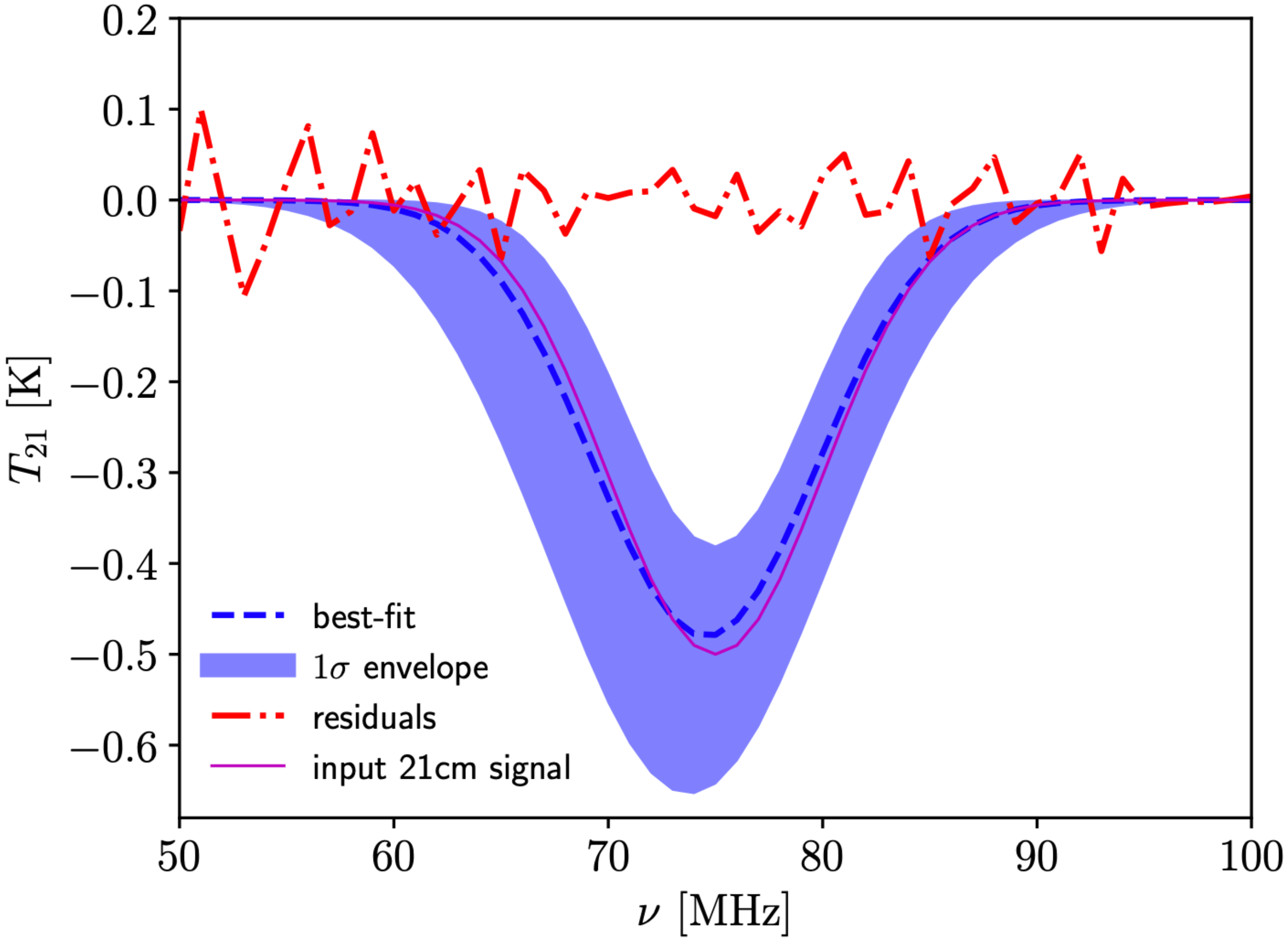




recovered global sky temperature:
24 snapshots/day,
each snapshot 100 hr integration time

ground-based 2D array:





$$T_{\rm FG}(\nu) = T_0 \left(\frac{\nu}{\nu_0}\right)^{\beta(\nu)}$$

$$\beta(\nu) = \sum_{i=0}^{N_a} a_i [\ln(\nu/\nu_0)]^i$$

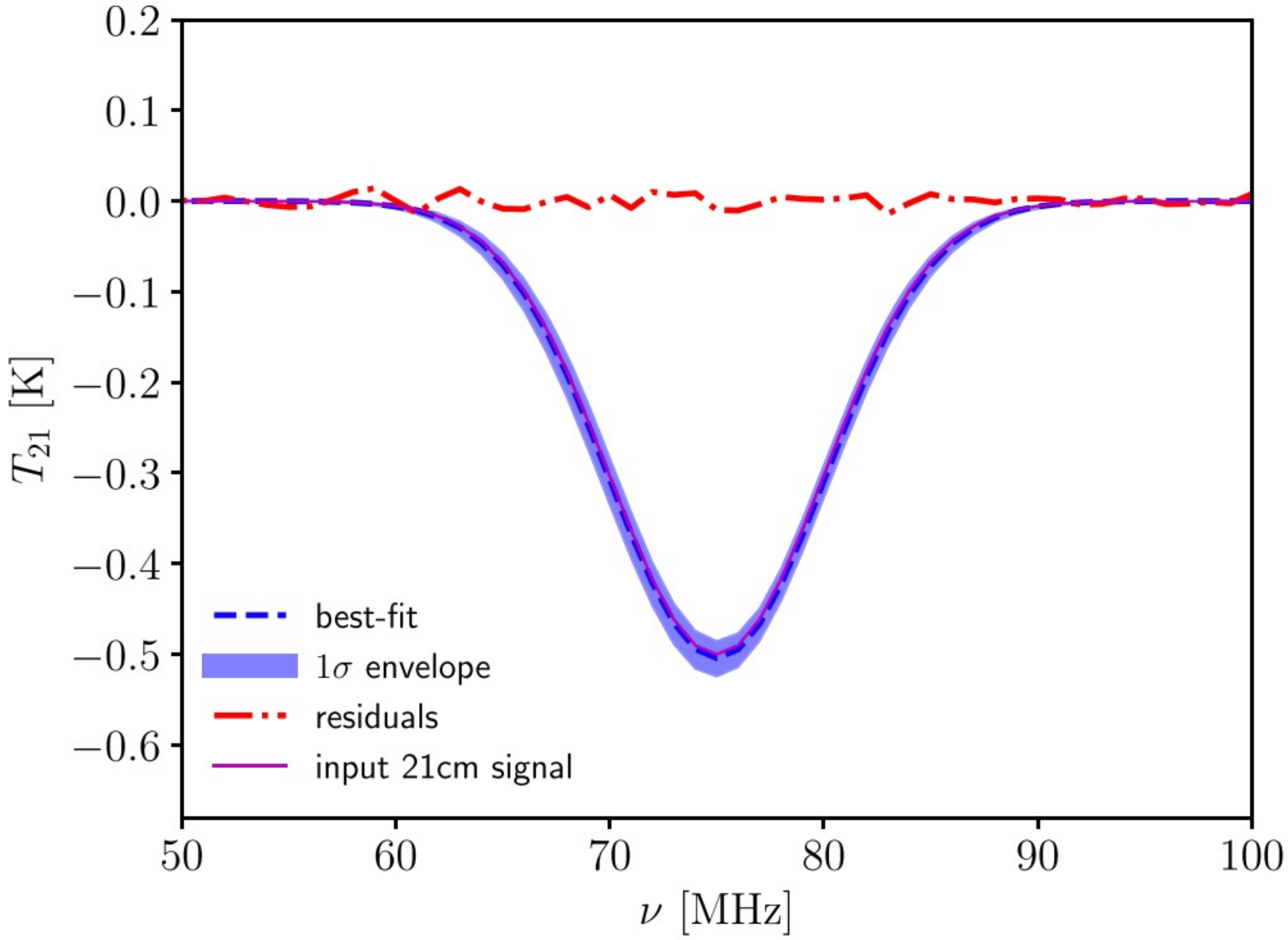
de OliveiraCosta et al. 2008

$$A = -0.49^{+0.04}_{-0.05}\,\mathrm{K}$$

$$\nu_{21} = 74.6^{+0.3}_{-0.3}\mathrm{MHz}$$

$$\sigma_{21} = 5.3^{+0.4}_{-0.4}\mathrm{MHz}$$

Ground-based 2D array
But combine the baselines of different snapshots together
Actually 3D baseline!



$$A = -0.505^{+0.007}_{-0.007}\,\mathrm{K}$$

$$\nu_{21} = 75.01^{+0.05}_{-0.05}\mathrm{MHz}$$

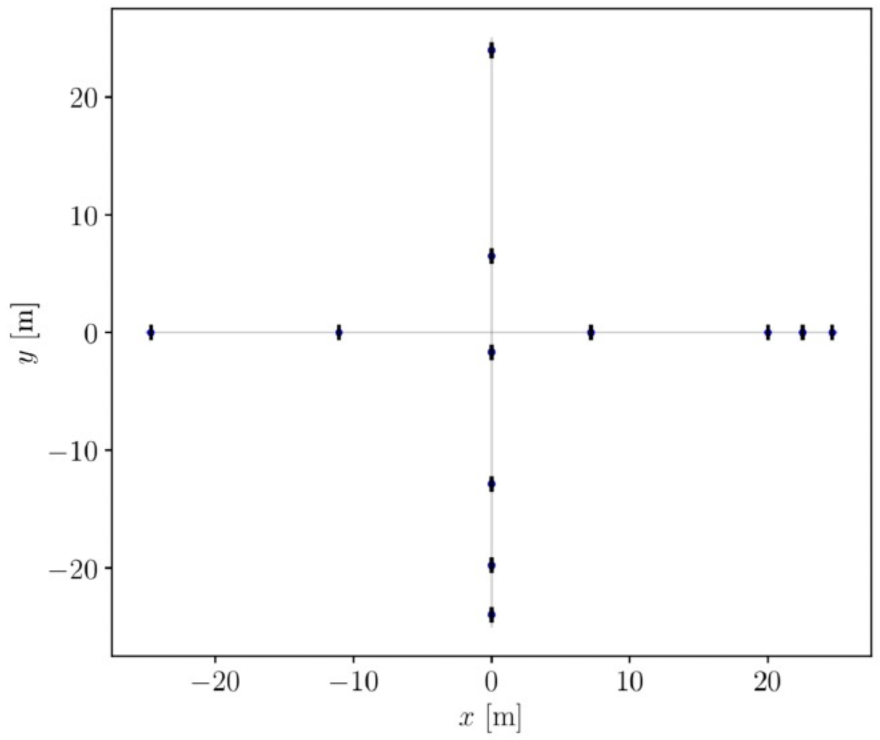
$$\sigma_{21} = 5.05^{+0.07}_{-0.07}\mathrm{MHz}$$

More better than that obtained by using only instantaneous 2D baselines,
We strongly recommended the 3D baseline!

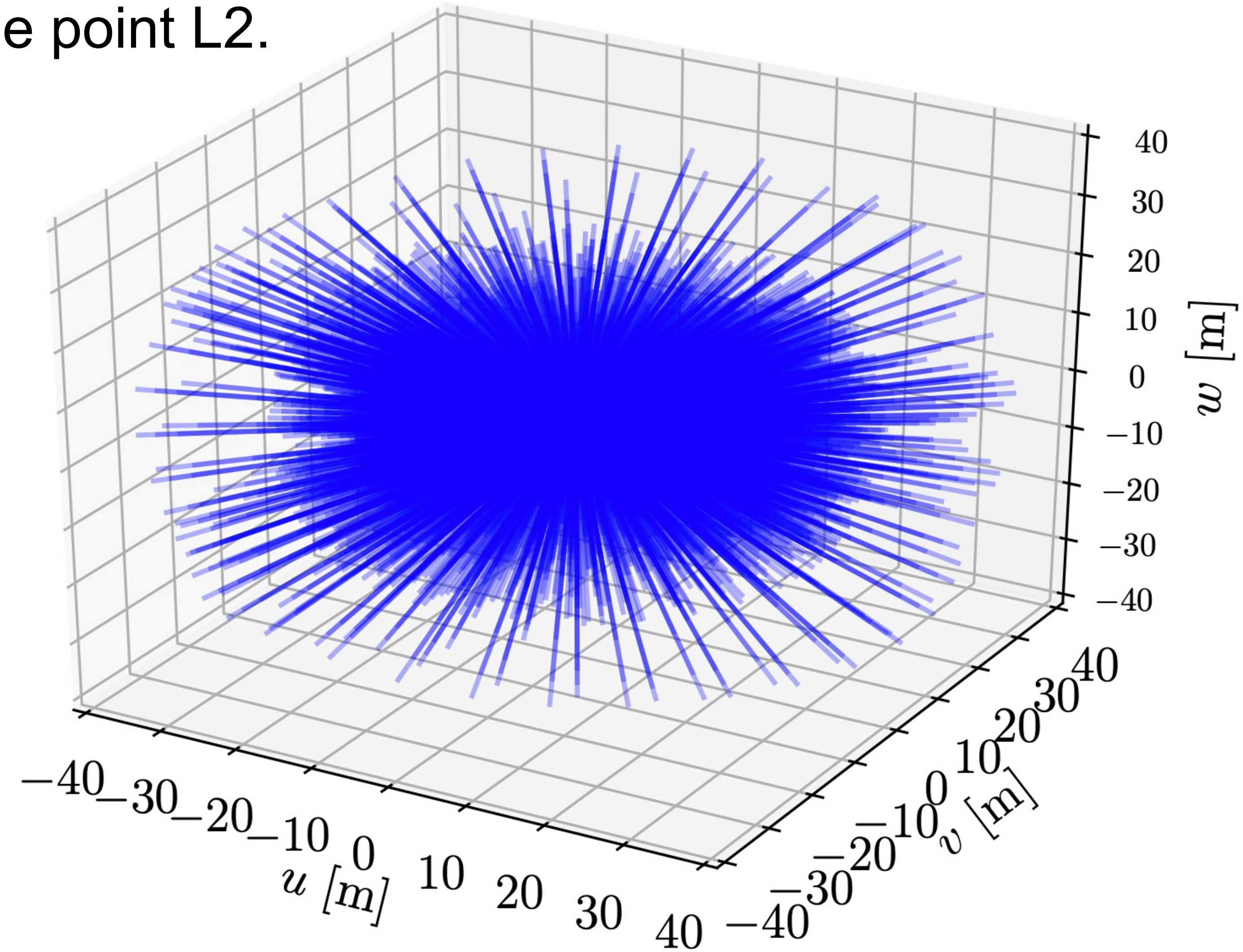
Simulated results: space array:

To overcome shielding and/or ground reflection effects, the best way is to build an array in space

For example at the Sun–Earth Lagrange point L2.

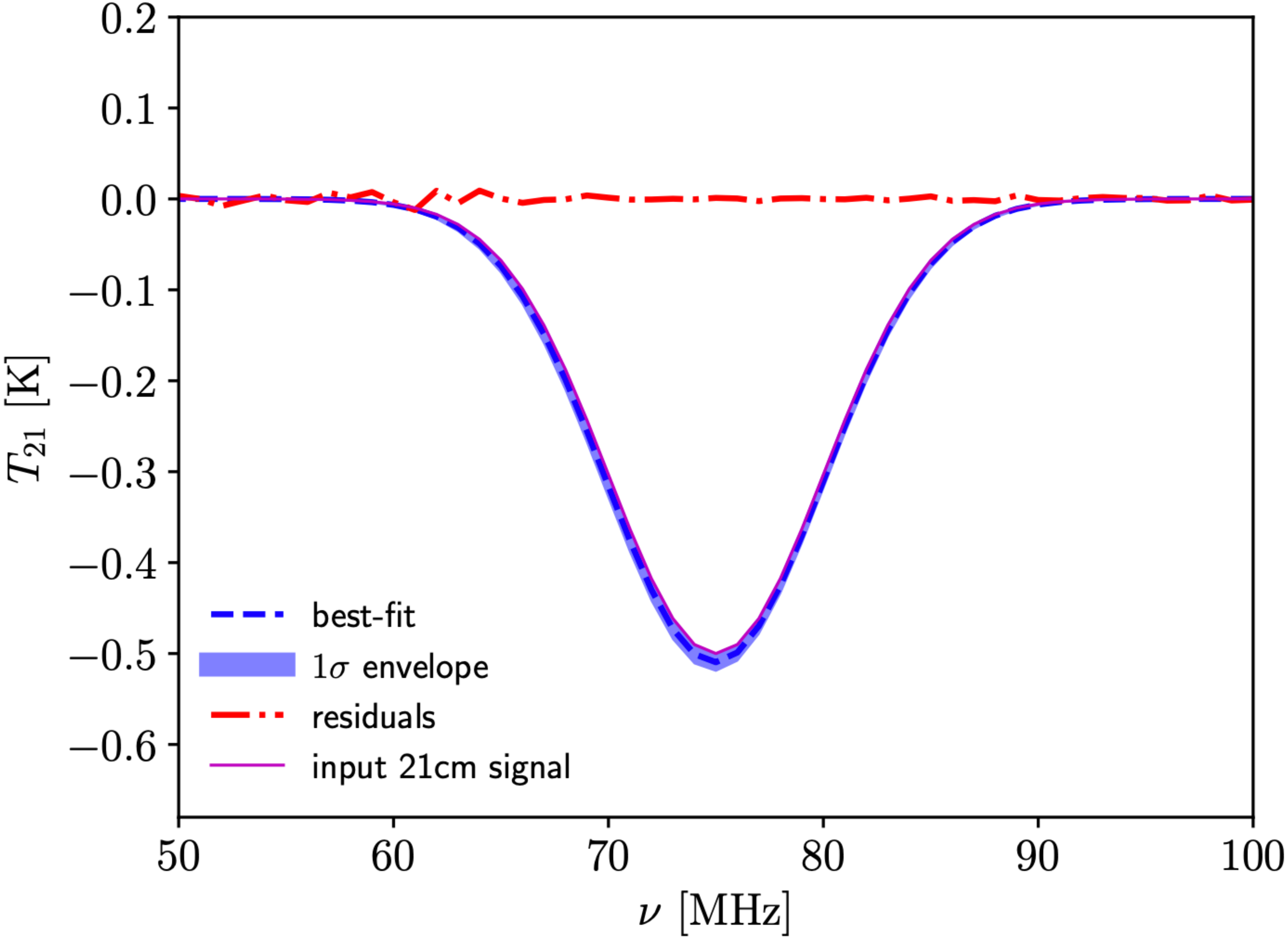


12 dipole antennas, cross-shaped



baselines formed via precessiom

Simulated results: space array:



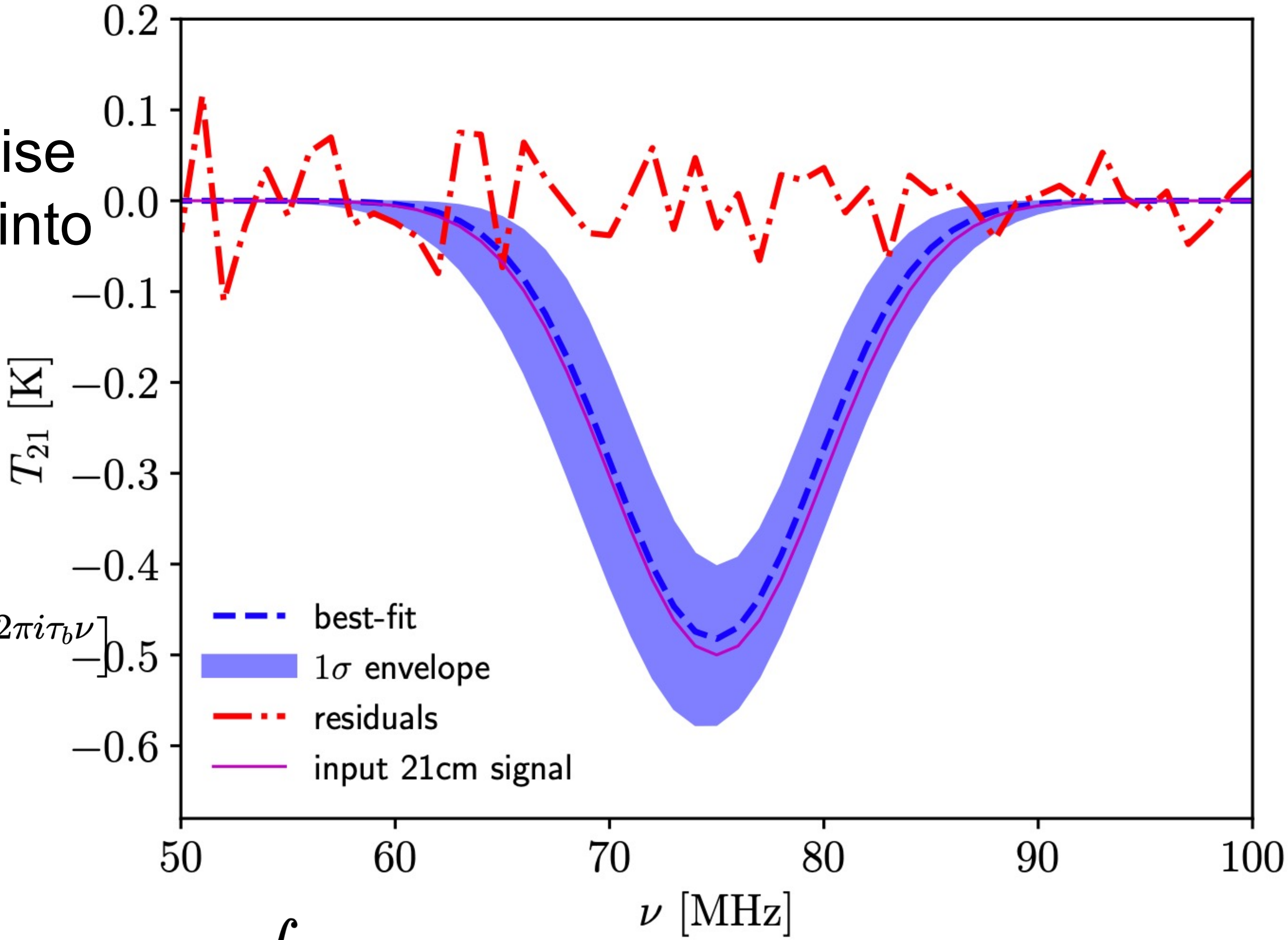
$$A = -0.509^{+0.004}_{-0.004}\ \mathrm{K}$$

$$\nu_{21} = 74.96^{+0.02}_{-0.02}\mathrm{MHz}$$

$$\sigma_{21} = 5.07^{+0.03}_{-0.03}\mathrm{MHz}$$

The cross-talk:

Origin from the internal noise of one antenna that leaks into another one, and the sky signal scattered by one antenna that received by another.



$$f_c(\nu, \boldsymbol{b}) = 0.01\left(\frac{b}{6\ \mathrm{m}}\right)^{-1}\left[1 + e^{2\pi i \tau_b \nu}\right]$$

$$\tau_b = 2b/c$$

$$V'_{12} = V_{12} + f_c^* \int d\Omega(\hat{\boldsymbol{n}}) B(\hat{\boldsymbol{n}}) T(\hat{\boldsymbol{n}}) + f_c \int d\Omega(\hat{\boldsymbol{n}}) B(\hat{\boldsymbol{n}}) T(\hat{\boldsymbol{n}}) + f_c f_c^* V_{12}^*$$

$$\boldsymbol{V}' = Q'\boldsymbol{a}$$

Summary:

1. The origin of EDGES 21cm absorption excess is still in debate

2. The monopole contribution is present in the visibility, we developed algorithm to recover the monopole from visibilities

3. We propose ground-based 2D, 3D and space interferometer arrays to detect the 21cm global spectrum.

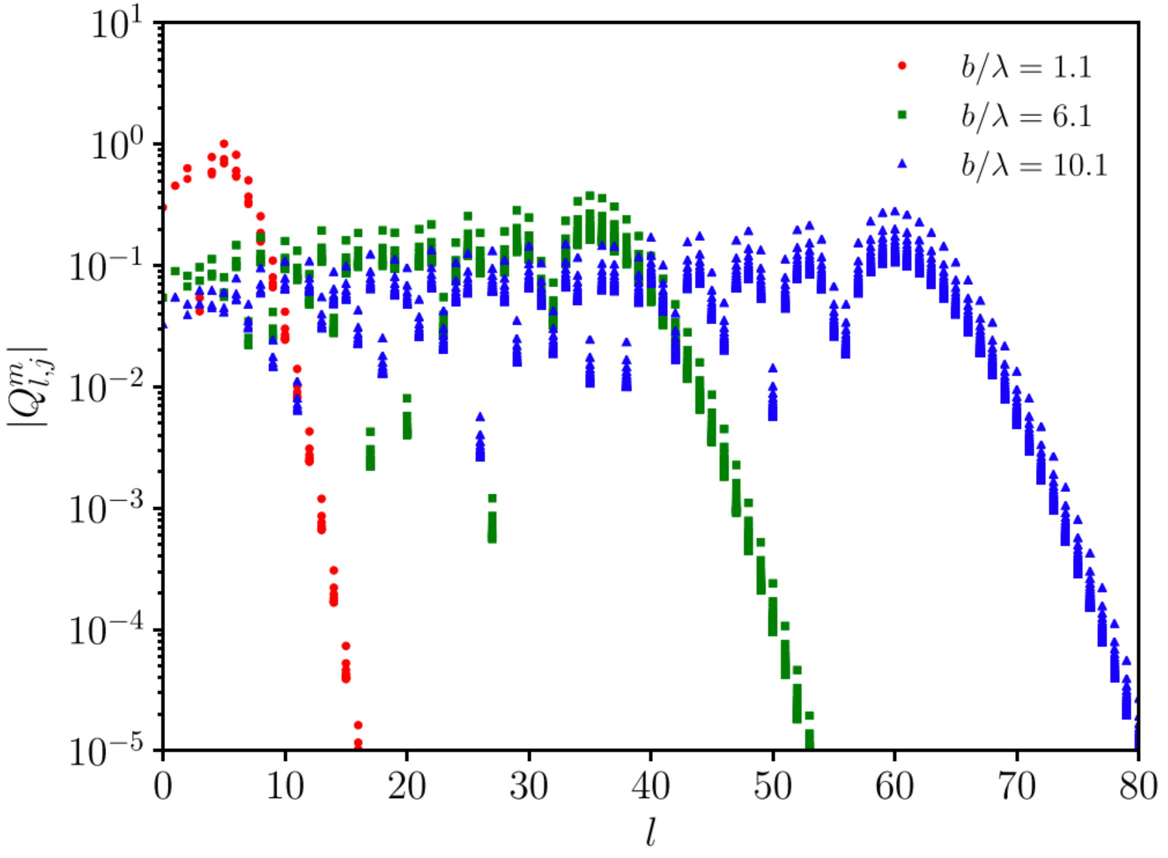
Thank You !

Appendix

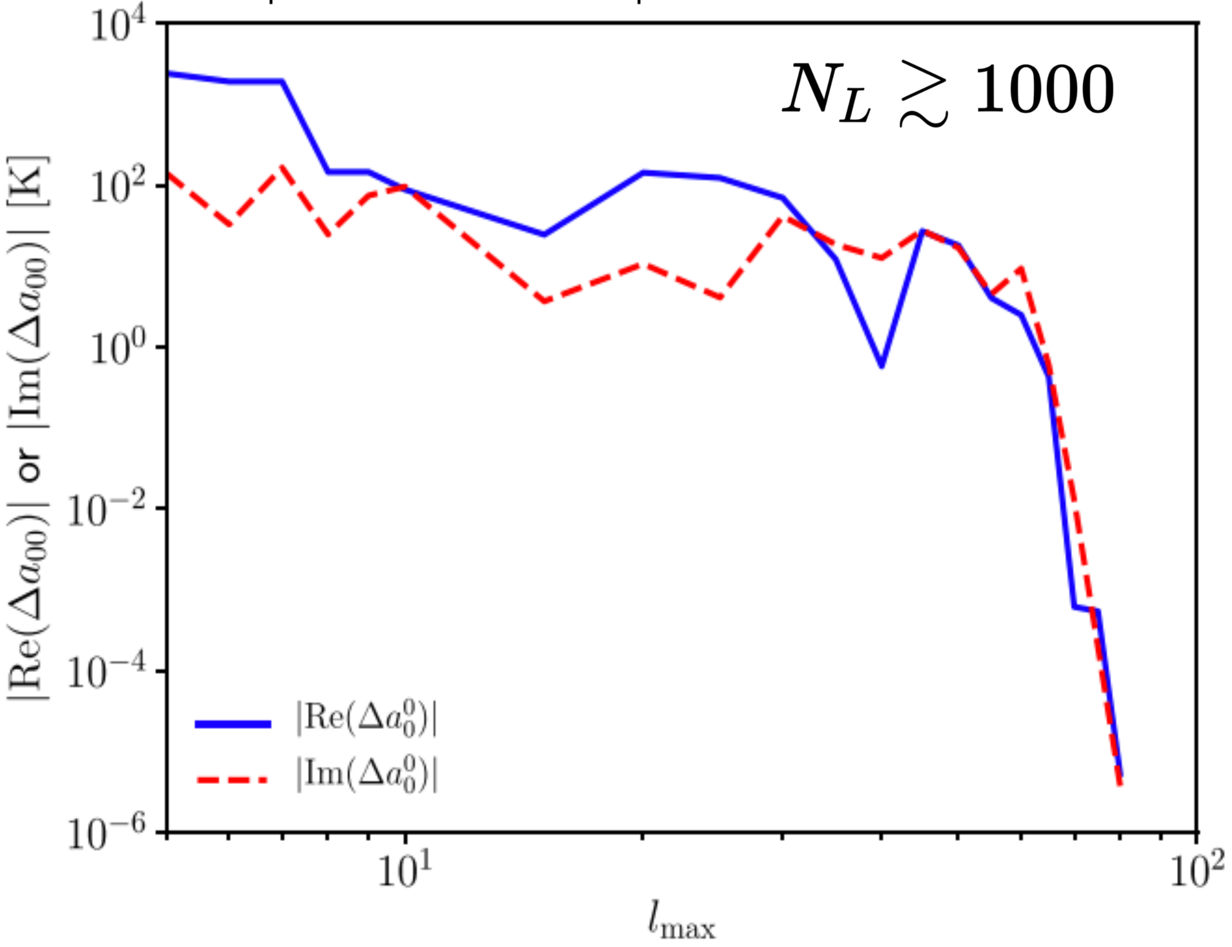
We consider baselines with $b \lesssim 10\lambda$

The interferometric measurement with a given baseline length b is only sensitive to modes up to: $l \sim 2\pi b/\lambda$

So, $l_{max} > 2\pi b_{\max}/\lambda \approx 60$

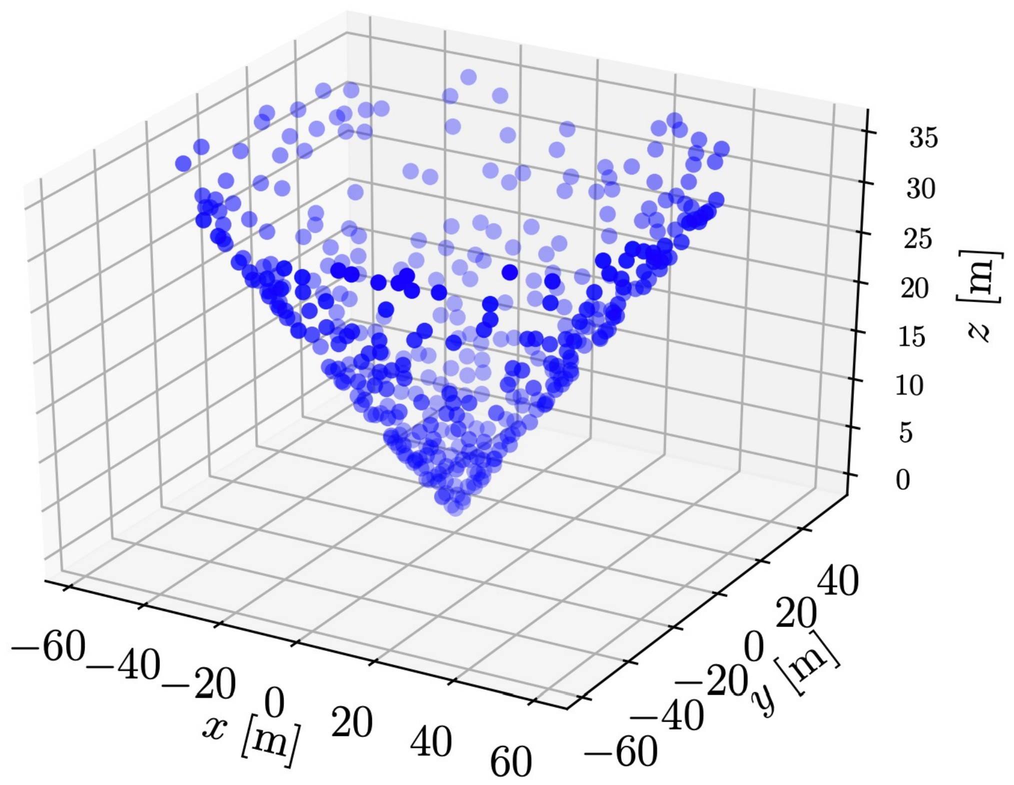


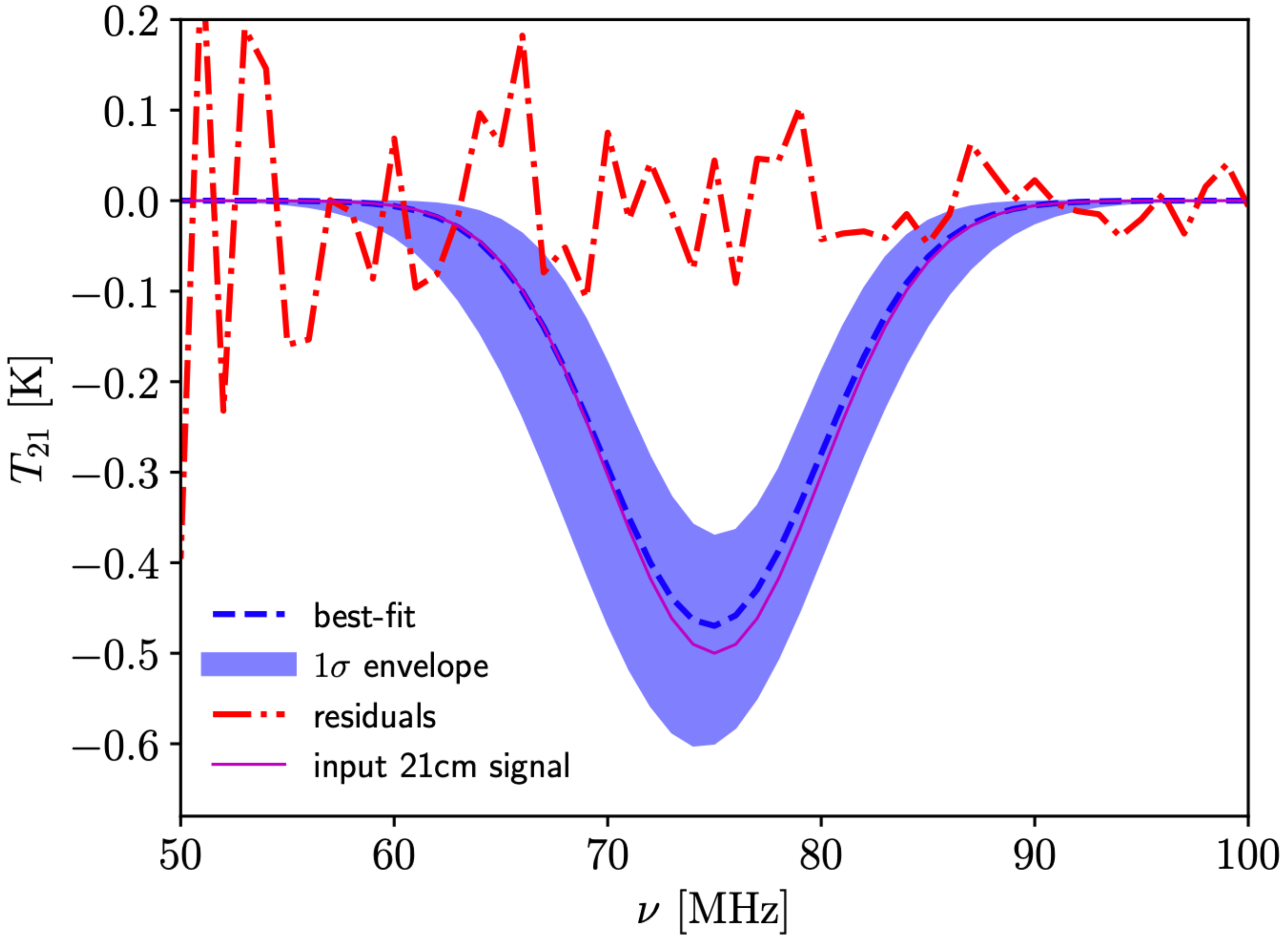
$\left|\hat{\tilde{a}}_0 - \mathrm{Re}(a^0_0)\right| / \mathrm{Re}(a^0_0) \lesssim 10^{-8}$



Simulated results: ground-based qusi-3D array:

qusi-3D array configuration





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