Needlet Karhunen-Loève Transform (NKL): A Method For Cleaning Foregrounds From 21cm Intensity Maps

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The problem at hand

- Our goal is to observe emission (or absorption) of the 21cm line of hydrogen.
- However, this desired signal is obscured by foregrounds.
- These foregrounds are in fact several orders of magnitude brighter than the HI signal.

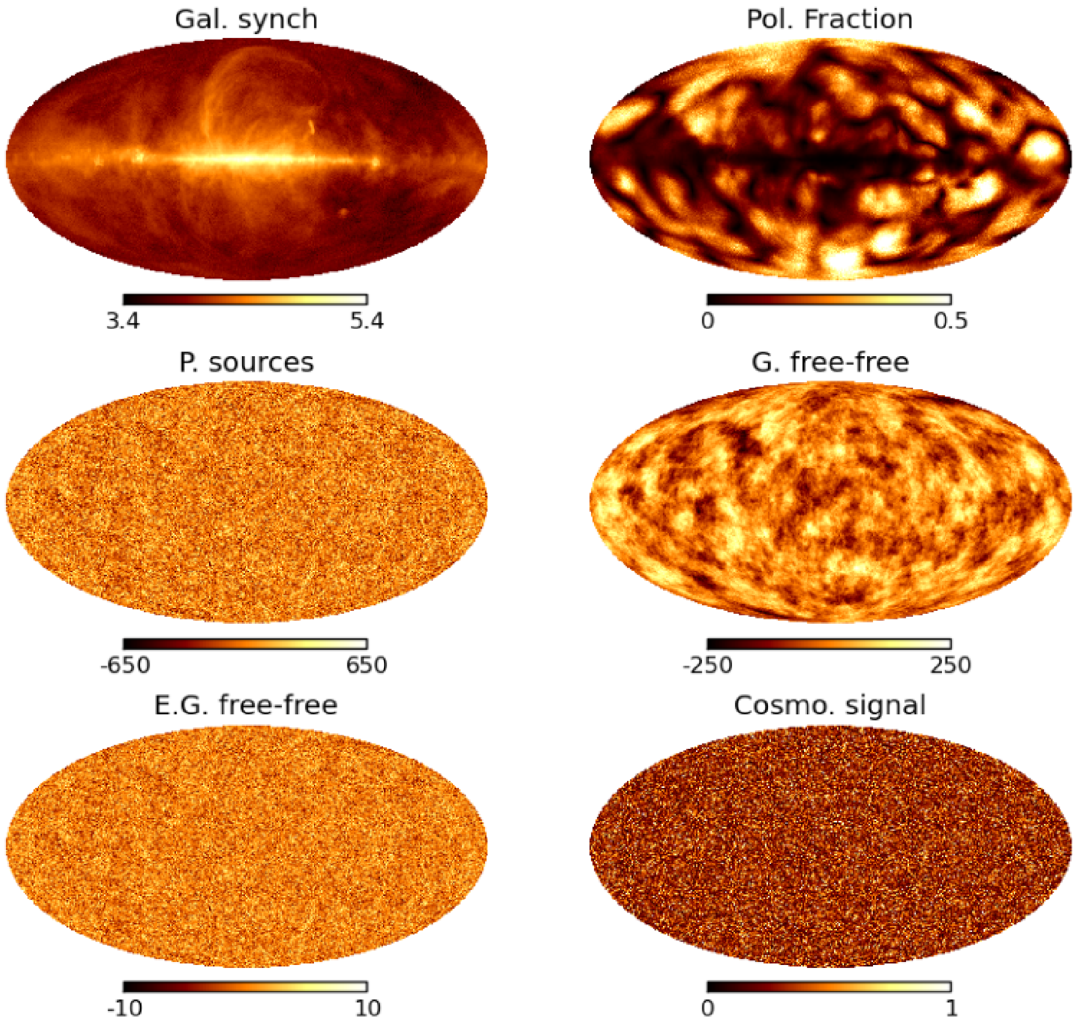


Figure 3. Full-sky maps of the different foregrounds and the cosmological signal for a frequency slice $\nu \sim 565$ MHz. Temperatures are given in mK (with the top left plot showing $\log_{10}(T_{\rm synch})$), except in the case of the polarized fraction (upper right panel), which is dimensionless.

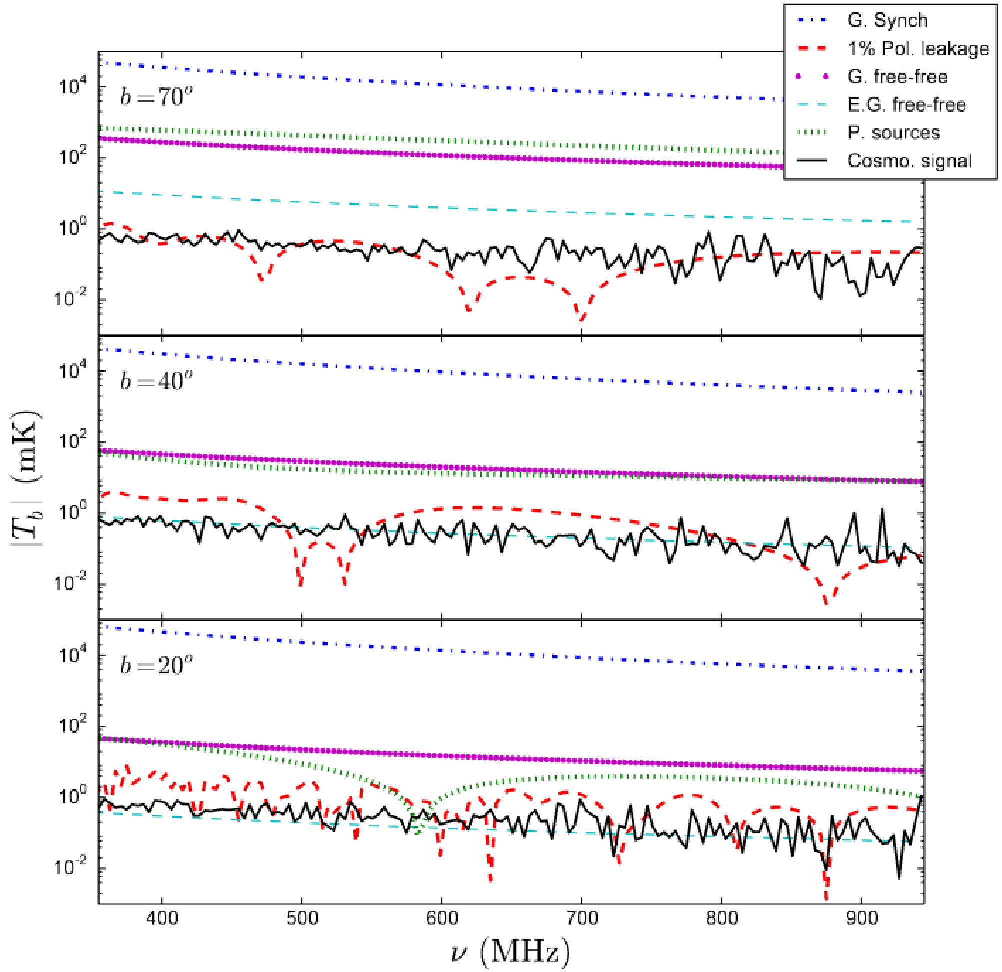
Simulated maps of various sources in the sky at 565MHz.

All foregrounds sources are at least an order of magnitude brighter than the 21cm signal, with galactic synchrotron emission being the most severe contaminant.

Note that the "polarized fraction" map refers to the polarized portion of the galactic synchrotron emission.

Figured borrowed from Alonso et al. 2014

Differences in frequency behavior



Fortunately, these various foreground components are expected to vary smoothly as a function of frequency.

This is obviously different than the HI signal (black), which appears more jagged.

The least smooth component of the foregrounds will be the polarized portion of galactic synchrotron radiation.

Mixing matrix formalism

- We have

$$\mathbf{x} = \mathbf{f} + \mathbf{s} + \mathbf{n}$$

foregrounds

signal

noise

$n_{ch} \times n_{pix}$ data matrix

- We then assume the foregrounds can expressed as

$$\mathbf{f} = \mathbf{A}\mathbf{S}$$

$\mathbf{A}$ is a $n_{ch} \times n_t$ "mixing matrix".

$\mathbf{S}$ is a $n_t \times n_{pix}$ "template matrix"

n_t is the number of templates.

A different viewpoint of the mixing matrix formalism...

- Let's consider just one matrix element, we have

$$f_{ij} = \sum_k A_{ik} S_{kj}$$

- Roughly speaking, we can think of the rows of **A** as basis functions and the columns of **S** as collections of coefficients.

$$f_j(\nu) = \sum_k A_k(\nu) S_{kj}$$

Some thoughts on this

- This matrix mixing formalism applies to many of the popular map-based foreground cleaning techniques (GMCA, PCA, ICA and GNILC).
- Techniques such as GMCA, PCA and ICA are typically conducted globally, i.e. there is one pair of **A** and **S** for the whole dataset.
- In other words, we are using the same functions to describe the frequency behavior of the foregrounds throughout the whole map.
- This is not ideal in the polarized case, since the frequency behavior of the foregrounds is expected to vary significantly throughout the map.

Some more thoughts

- In addition to varying with line-of-sight, the foreground chromaticity is also expected to vary as a function of angular scale, with large scales being more corrupted than small scales.
- These features of the foregrounds motivate the development of methods that clean the foregrounds in a more local way.
- GNILC satisfies this goal, cleaning the foregrounds one subsection of line-of-sight / angular scale space at a time.
- A shortcoming of GNILC is that it only considers correlations along the line of sight.
- However, we know that different lines of sight in the map ought to be correlated. In fact, we even have priors for these correlations.
- This motivates the development of a new foreground cleaning method.

Desirable properties for a new foreground cleaning method

1.) cleans locally in line-of-sight / angular scale space. At the very least, it should not use the same **A** for all locations in the locations in the map.

2.) Considers the whole map when generating **A** and **S** for some neighborhood.

Our NKL technique happens to have these properties!

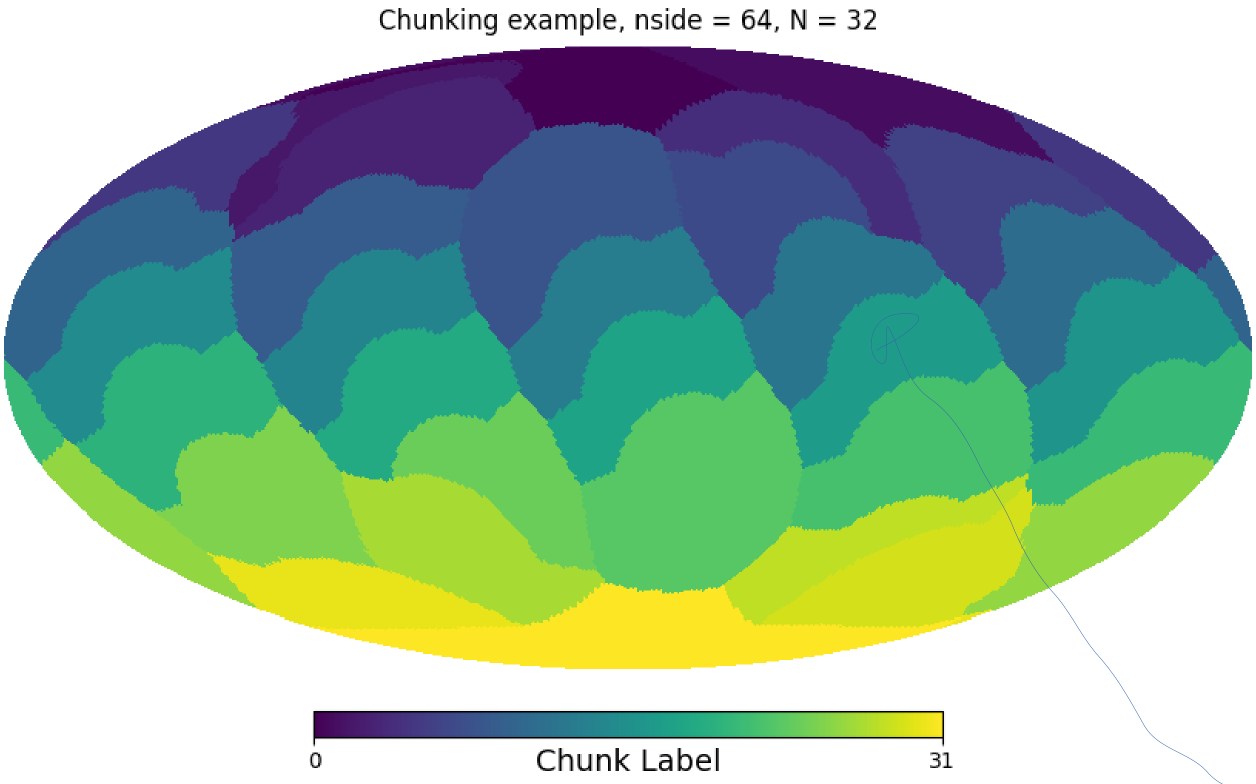
The NKL technique put simply

1.) Send maps into the needlet domain. (essentially a set of bandpass function in l-space)

2.) **Generate a FG approximation** in the needlet domain (can be done using PCA or DAYENU methods)

3.) **Partition the needlet coefficients** into N “chunks”. These “chunks” are regions of pixel-space.

4.) Use the foreground approximation, along with priors on the signal and noise in order to **perform a Karhunen-Loève transform. This KL transform will clean the foregrounds from the data.**



An example of the “chunks” mentioned in the previous slide.

nside = 64

N = 32 chunks

Example of a chunk

Karhunen-Loève Transform

$$\mathbf{C}_{FG}\mathbf{\Phi} = \mathbf{C}_S\mathbf{\Phi}\mathbf{\Lambda}. \qquad (5)$$

In this formula, $\mathbf{\Phi}$ is a matrix of eigenvectors and $\mathbf{\Lambda}$ is a diagonal matrix of eigenvalues. It turns out that the eigenvectors $\mathbf{\Phi}$ obtained are a solution to the optimization problem [31]

$$\max_{\mathbf{\Phi}} \mathbf{tr}(\mathbf{\Phi}^T\mathbf{C}_{FG}\mathbf{\Phi})$$
$$\text{subject to } \mathbf{\Phi}^T\mathbf{C}_S\mathbf{\Phi} = \mathbf{I}. \qquad (6)$$

The KL transform seeks to find modes that have the largest ratio of foreground to signal possible.

$$\mathbf{S} = \mathbf{P}_s\mathbf{X}.$$

Foregrounds dominated rows of $P = \Phi^\dagger$

$$\mathbf{A} = \mathbf{P}_s^{-1},$$

Columns of P^{-1} corresponding to foreground dominated modes.

This becomes a standard eigenvalue problem when

$$\mathbf{C}_S = \mathbf{I}$$

KL, part 2

- Due to the chunking, we find that **each chunk of the map has its own mixing matrix**.
- Moreover, these **mixing matrices are derived in a global way.**
- For instance, the mixing matrix of chunk 10 will depend the map as a whole, rather than just chunk 10 in isolation.
- Thus, **NKL satisfies has the desirable properties described earlier in this talk**.

Test cases

Parameter	Value
D	13.5 m
T_{inst}	20 K
f_{sky}	1
t_{obs}	40000 hrs
N_{dishes}	64
n_{ch}	256
$[\nu_{min}, \nu_{max}]$	[980, 1080 MHz] and [400, 500 MHz].
$\Delta\nu$	0.390625 MHz

TABLE I. Parameters describing the hypothetical telescope used in this paper.

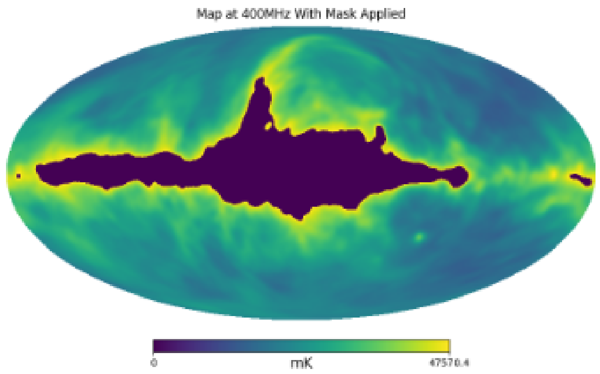
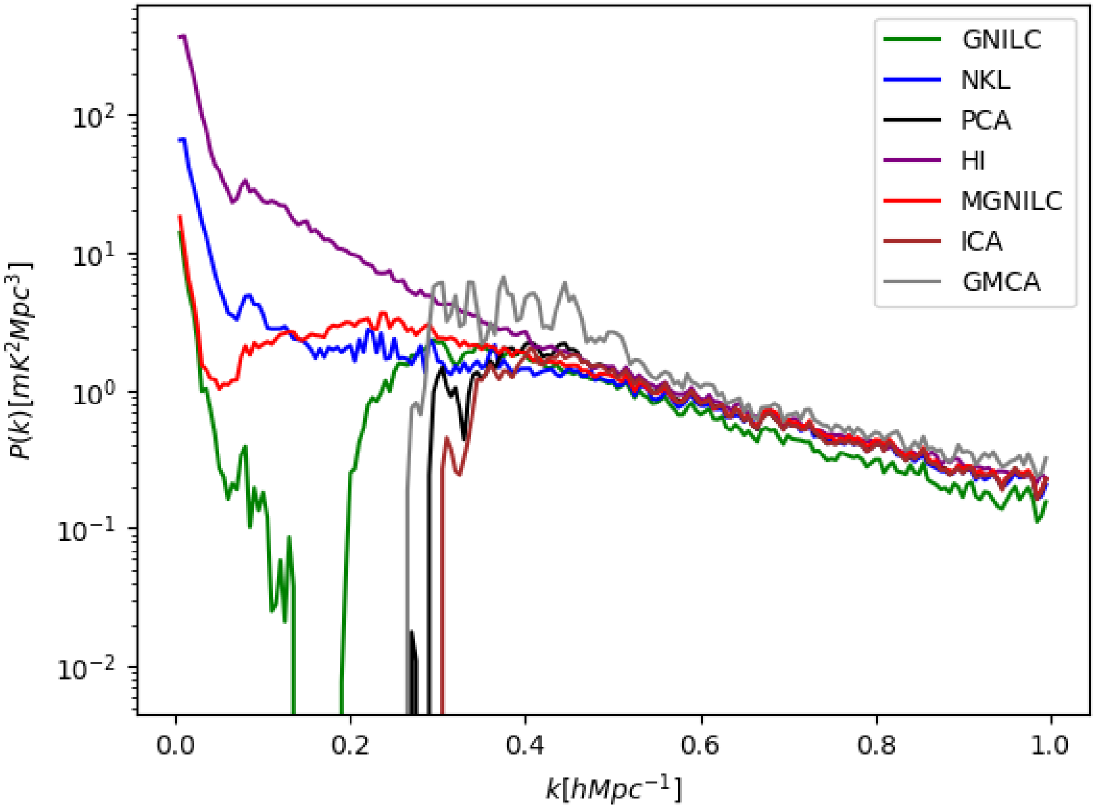


FIG. 2. An illustration of the mask used for tests of the foreground removal methods in the high redshift case. The mask has been applied to this map, which includes simulated foregrounds, signal and noise.

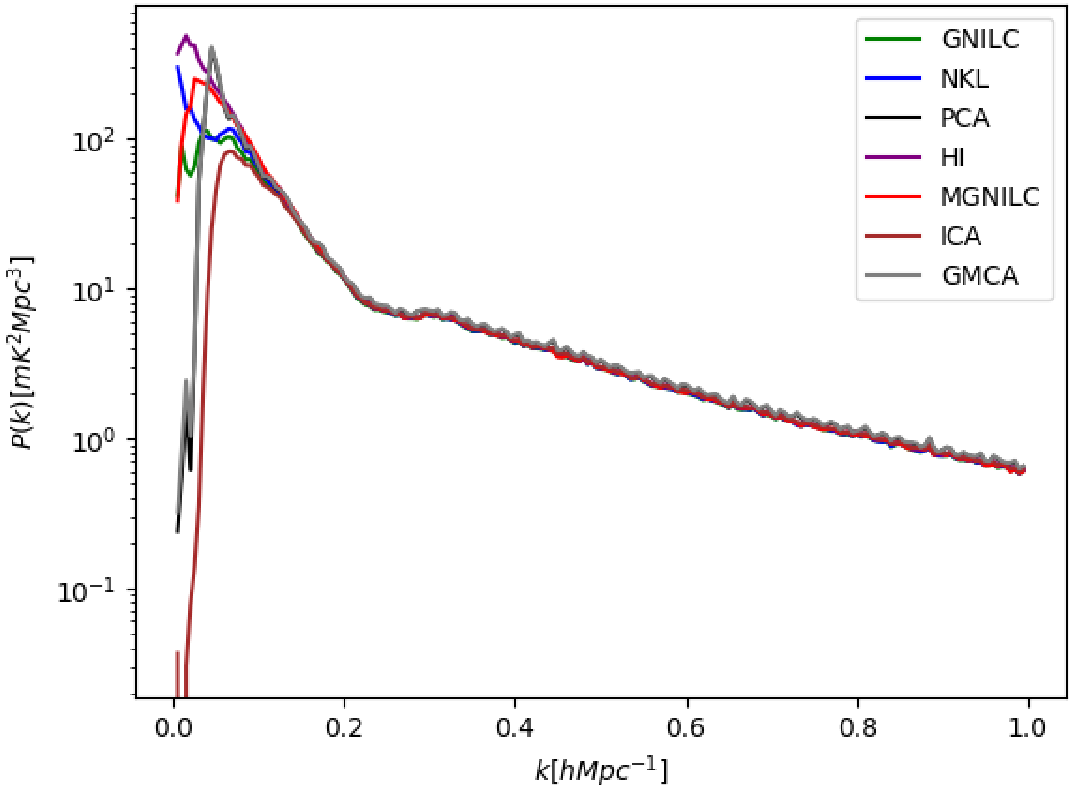
Performed two tests, one from 980 to 1080MHz and the other from 400 to 500MHz.

Galactic synchrotron radiation was the only foreground considered. Polarized emission was included.

Results at higher redshift



Results at lower redshift



Conclusions

- The nature of polarized galactic synchrotron radiation motivates the use of a more local foreground cleaning method.
- The existence of HI correlations between different lines of sight motivates considering the whole map while cleaning.
- NKL satisfies both of these properties, and provided improvements in performance during our tests.
- We are also working on applying similar ideas to visibility space.

- Stay tuned for a publication on this work!

Acknowledgments

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