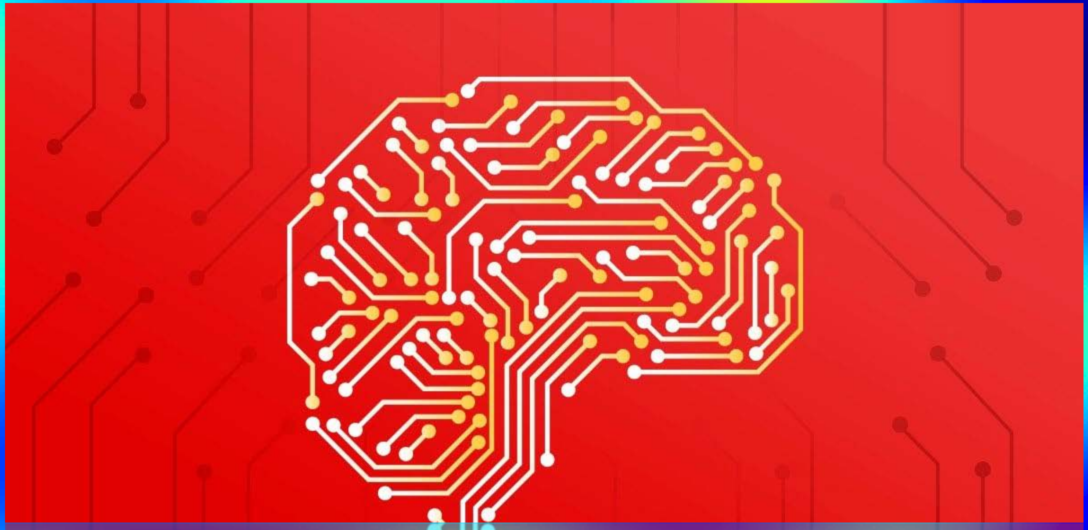

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Map Reconstruction of Radio Observations with Conditional Invertible Neural Networks

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In collaboration with S.F. Zuo and H.L. Zhang
arxiv:2306.09217; Research in Astronomy and Astrophysics

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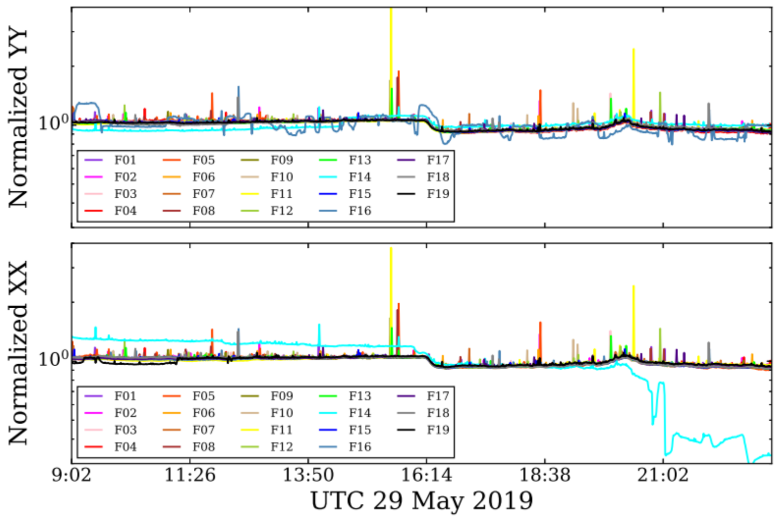
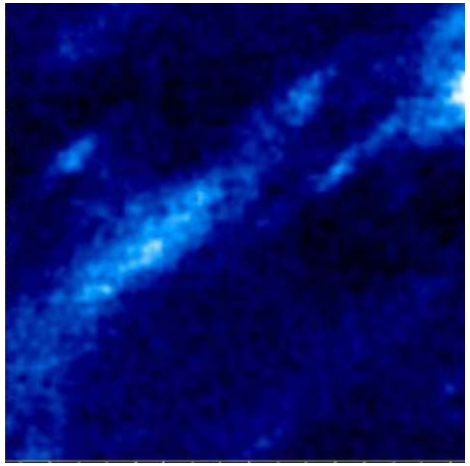
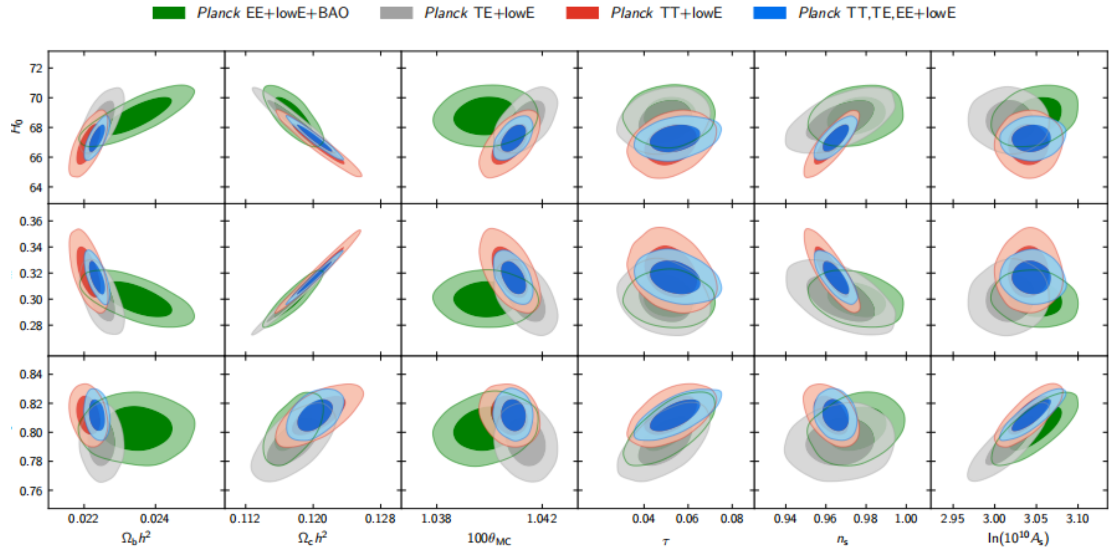
1.1 Necessity of map-making

- Map-making is a crucial step in radio observations, bridging the gap between the collected TODs and scientific analysis
- important to produce pixelized maps from TOD, with as much accuracy as possible

time-ordered data(TOD) → Sky Map → Data Analysis





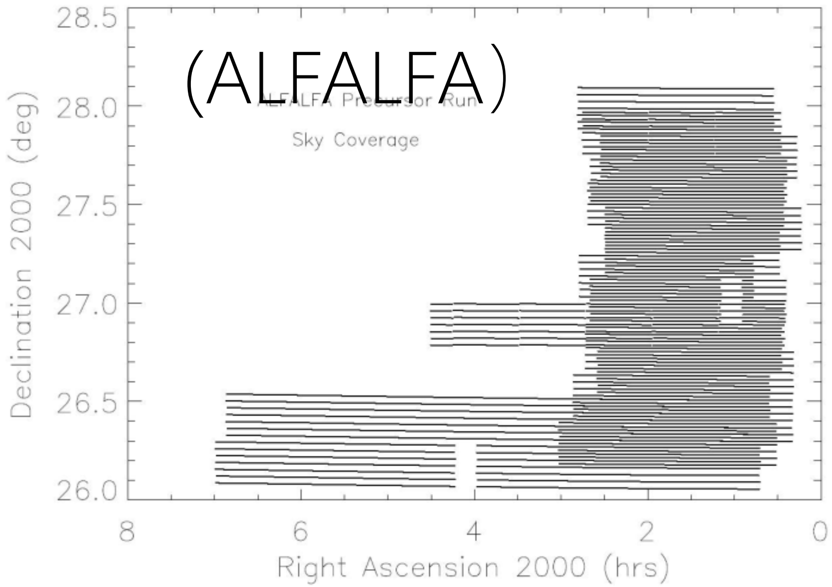


Hu 2021

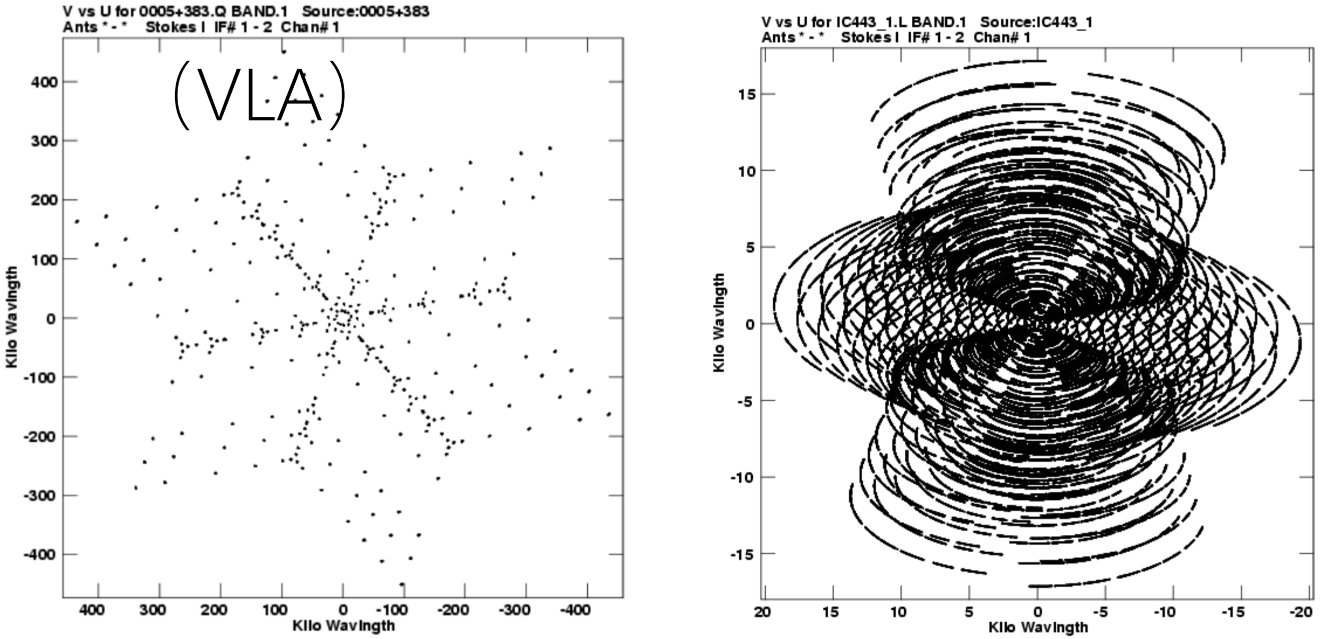
Planck 2018

1.2 Issues in map-making

- the TOD's sampling is uneven and irregular in practice
- **Map-making usually is an ill-posed inverse problem**—observational effects such as scan strategies, noise, complex geometry of the field and data excision like RFI flagging
- unbiased estimate with minimal variance is a big challenge



Scanning pattern of a single-aperture telescope



uv-coverage from interferometric observation

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2.1 Traditional estimator

- **The concept of Map-making is to apply a constructed linear operator W on data**
- $\vec{d} = A\vec{m} + \vec{n}$:

$$\hat{m} = W\vec{d}$$

MAP-MAKING METHODS

Number	Method	Specification
1	Generalized *COBE*	$\mathbf{W} = [\mathbf{A}^t\mathbf{M}\mathbf{A}]^{-1}\mathbf{A}^t\mathbf{M}$
2	Bin averaging	$\mathbf{W} = [\mathbf{A}^t\mathbf{A}]^{-1}\mathbf{A}^t$
3	*COBE*	$\mathbf{W} = [\mathbf{A}^t\mathbf{N}^{-1}\mathbf{A}]^{-1}\mathbf{A}^t\mathbf{N}^{-1}$
4	Wiener 1	$\mathbf{W} = \mathbf{S}\mathbf{A}^t[\mathbf{A}\mathbf{S}\mathbf{A}^t + \mathbf{N}]^{-1}$
5	Wiener 2	$\mathbf{W} = [\mathbf{S}^{-1} + \mathbf{A}^t\mathbf{N}^{-1}\mathbf{A}]^{-1}\mathbf{A}^t\mathbf{N}^{-1}$
6	Saskatoon	$\mathbf{W} = [\eta\mathbf{S}^{-1} + \mathbf{A}^t\mathbf{N}^{-1}\mathbf{A}]^{-1}\mathbf{A}^t\mathbf{N}^{-1}$
7	TE96	$\mathbf{W} = \Lambda\mathbf{S}\mathbf{A}^t[\mathbf{A}\mathbf{S}\mathbf{A}^t + \mathbf{N}]^{-1}$, $(\mathbf{W}\mathbf{A})_{ii} = 1$
8	TE97	$\mathbf{W} = \Lambda[\eta\mathbf{S}^{-1} + \mathbf{A}^t\mathbf{N}^{-1}\mathbf{A}]^{-1}\mathbf{A}^t\mathbf{N}^{-1}$, $(\mathbf{W}\mathbf{A})_{ii} = 1$
9	Maximum probability	Nonlinear method if non-Gaussian
10	Maximum entropy	Nonlinear method

Tegmark, M (1997)

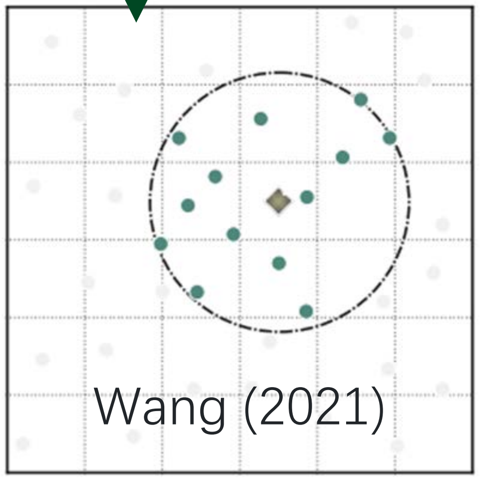
- The inverse of the matrix m may not exist, and therefore W may not exist
- Even if W exists, the computational complexity of the inverse operation can reach $O(N^3)$; computationally intractable if $N_p \sim 10^6$
- RFI and ⋯ subtractions in data preprocessing, leading to a degeneracy in the estimated map
- An accurate estimate of the noise is required, i.e., the covariance matrix N of the noise needs to be known
- The error estimate for each pixel is rather difficult

2.2 Traditional estimator: gridding approach

- The gridding: based on a direct weighted interpolation of TOD

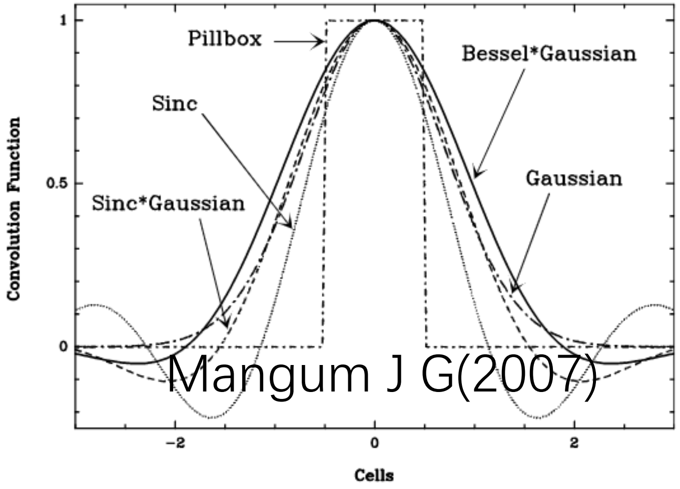
Convolution of TOD in a certain region

$$R_{i,j}\left(\alpha_{i,j},\delta_{i,j}\right)=\frac{1}{W_{i,j}}\sum_{n}R_n\left(\alpha_n,\delta_n\right)\mathrm{w}\left(\alpha_{i,j},\delta_{i,j};\alpha_n,\delta_n\right)$$



Wang (2021)

w: a convolution kernel

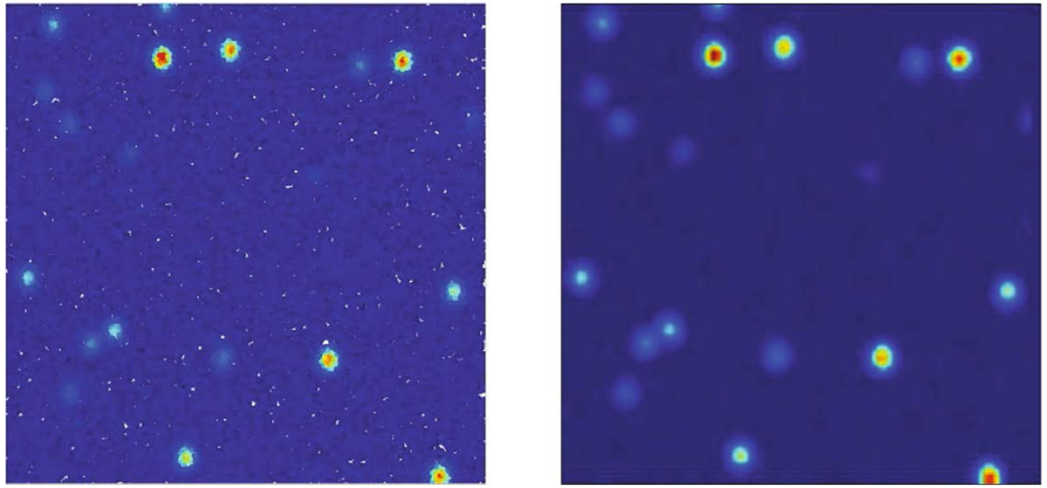


Mangum J G(2007)

Drawback:

- pixel-level-error estimate is difficult
- Gridding will further reduce the map resolution

$$\sigma_{gridded}=\sqrt{\sigma_{kernel}^2+\sigma_{data}^2}$$



Before and after gridding Luo(2018)

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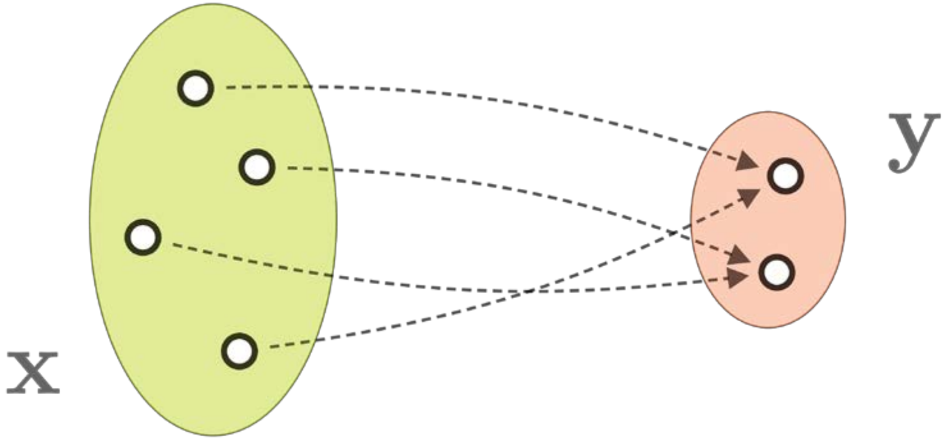
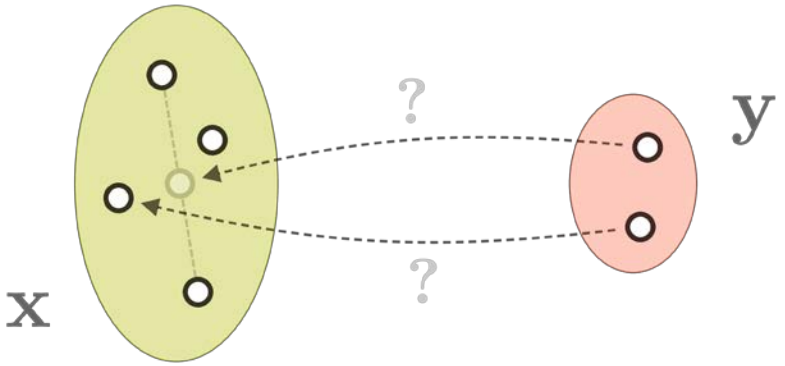
3.1 Inverse problem

- Inverse problems are common in scientific research, where observations are utilized to infer physical parameters (i.e., map here)
- The forward modeling process (map → TOD) is well understood
- TOD → map is of an inverse problem

Ambiguous Inverse Problems (ill-posed)

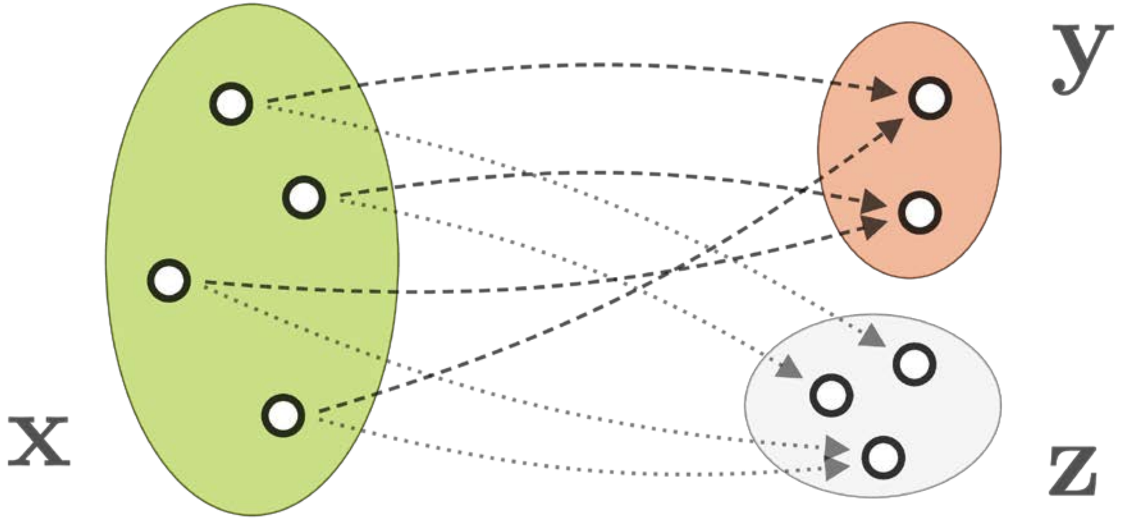




https://hci.iwr.uniheidelberg.de/vislearn/inverse-problems-invertible-neural-networks/

- one may apply statistical inference techniques to express the ambiguities in form of conditional probabilities $p(x|y)$
- classical Bayesian methods become very expensive even for moderate real-world problems

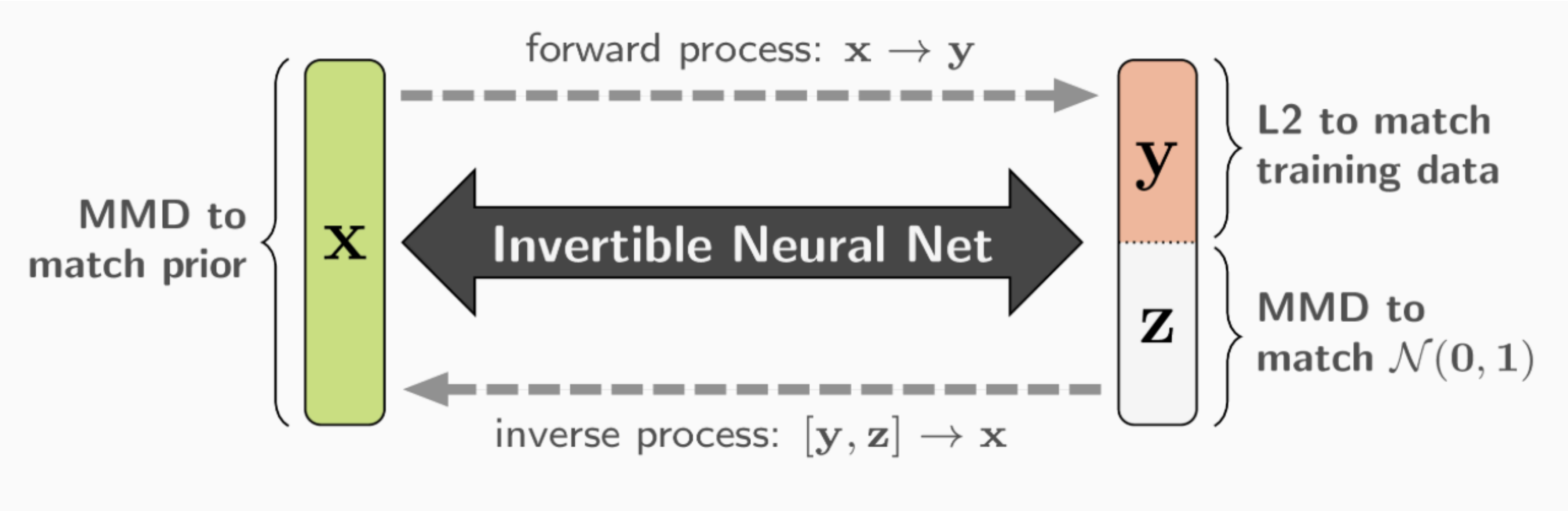
3.1 Resolving the ambiguity



Intuitively, the ambiguity of inverse mapping is transformed into $p(z)$.

- Bijective mapping: introducing additional latent variables, z, to preserve the information that would otherwise be lost during the forward process
- The mapping $x \leftrightarrow [y, z]$ becomes a one-to-one correspondence (well posed)
- In other words, $p(x|y)$ has been reparametrized into a deterministic function $x = f(y, z)$ with adding variable z

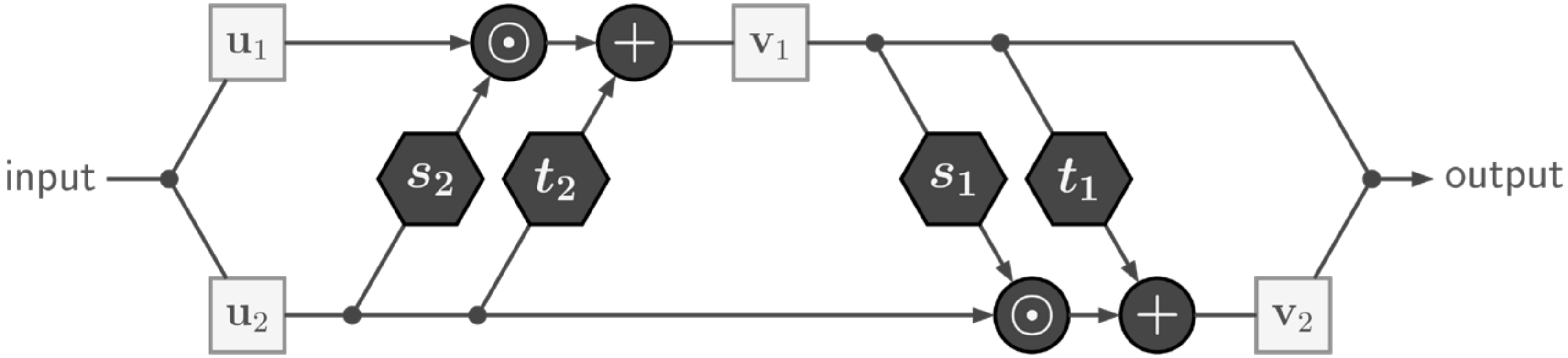
3.2 Training process



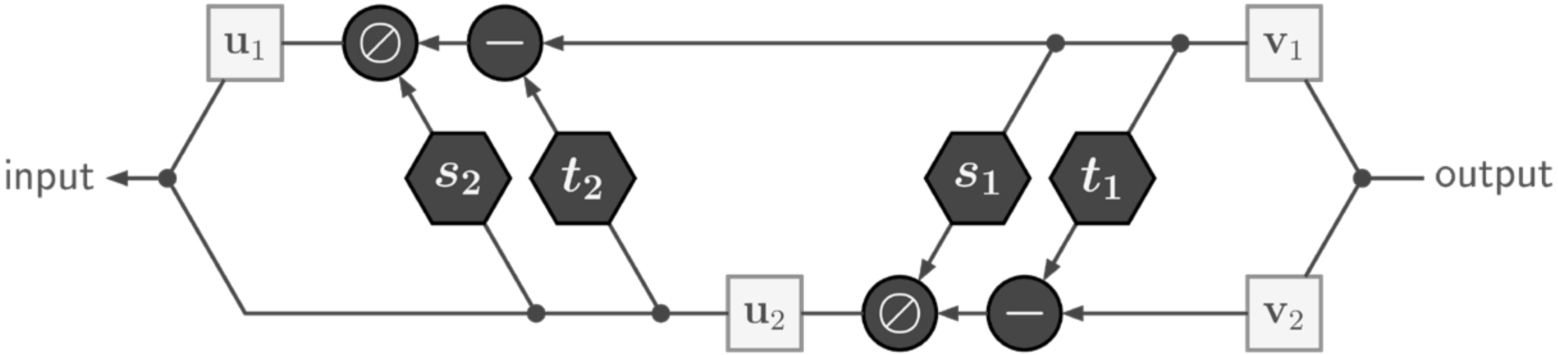
- train INN to solve the well-posed forward process $x \to y$ in a supervised manner, instead of the ill-posed inverse process
- the latent variables z to be independent of y, and to follow an easy-to-sample-from distribution, like $\mathcal{N}(0,1)$.
- use L2 (to match the data)+ a Maximum Mean Discrepancy (MMD) loss (to match the normal distribution) for training INN

3.3 Basic building block of INN

the *affine coupling layer* popularized by the **Real NVP** model.



Inverse of the whole affine coupling layer: recover $[\mathbf{u}_1, \mathbf{u}_2]$ from $[\mathbf{v}_1, \mathbf{v}_2]$



To construct deep invertible networks, just chain these layers like **ResNet blocks.**

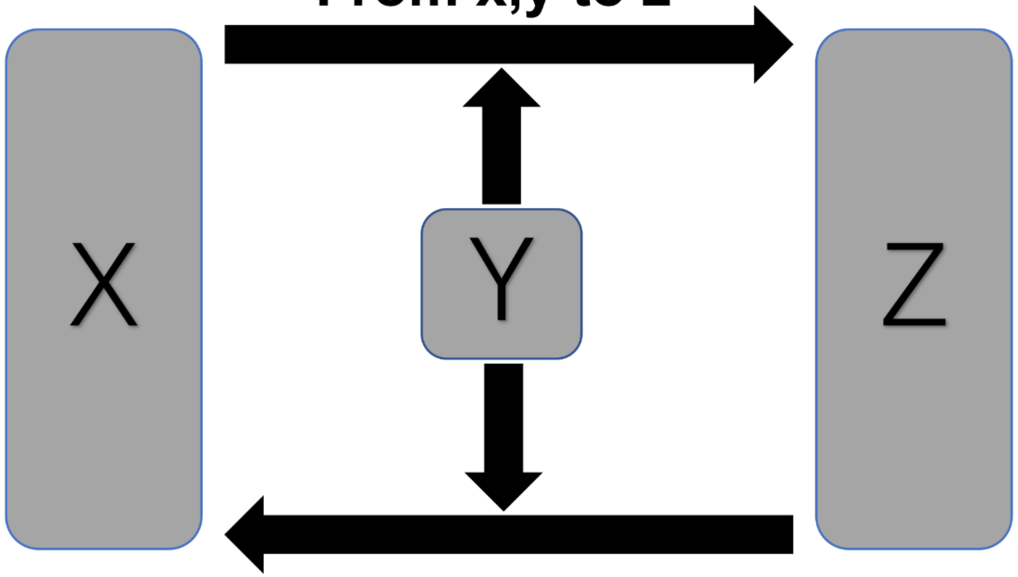
3.4 Conditional Invertible Neural Networks

- cINN: a modification of INN, enabling simpler training

- under the condition (y), the distribution of x is obtained by sampling z from Gaussian distribution; naturally obtain the error of x
- changed the role of y on the input side, while introducing the advantage of conditional input

$$z = f(x|y, \theta)$$

Forward Modeling:
From x,y to z



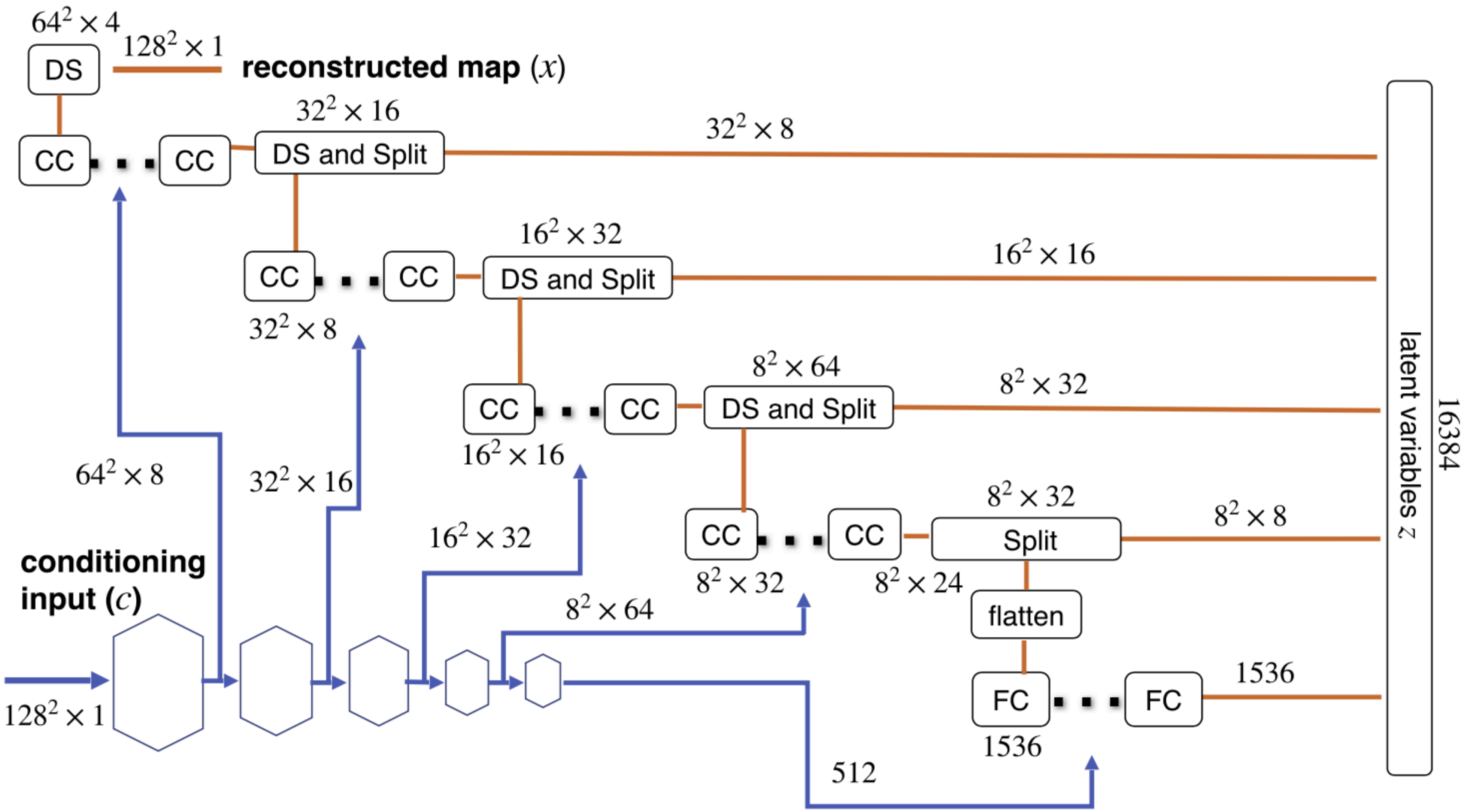
Backward Sampling:
From z,y to x

$$x = f^{-1}(z|y, \theta)$$

Ardizzone (2019)

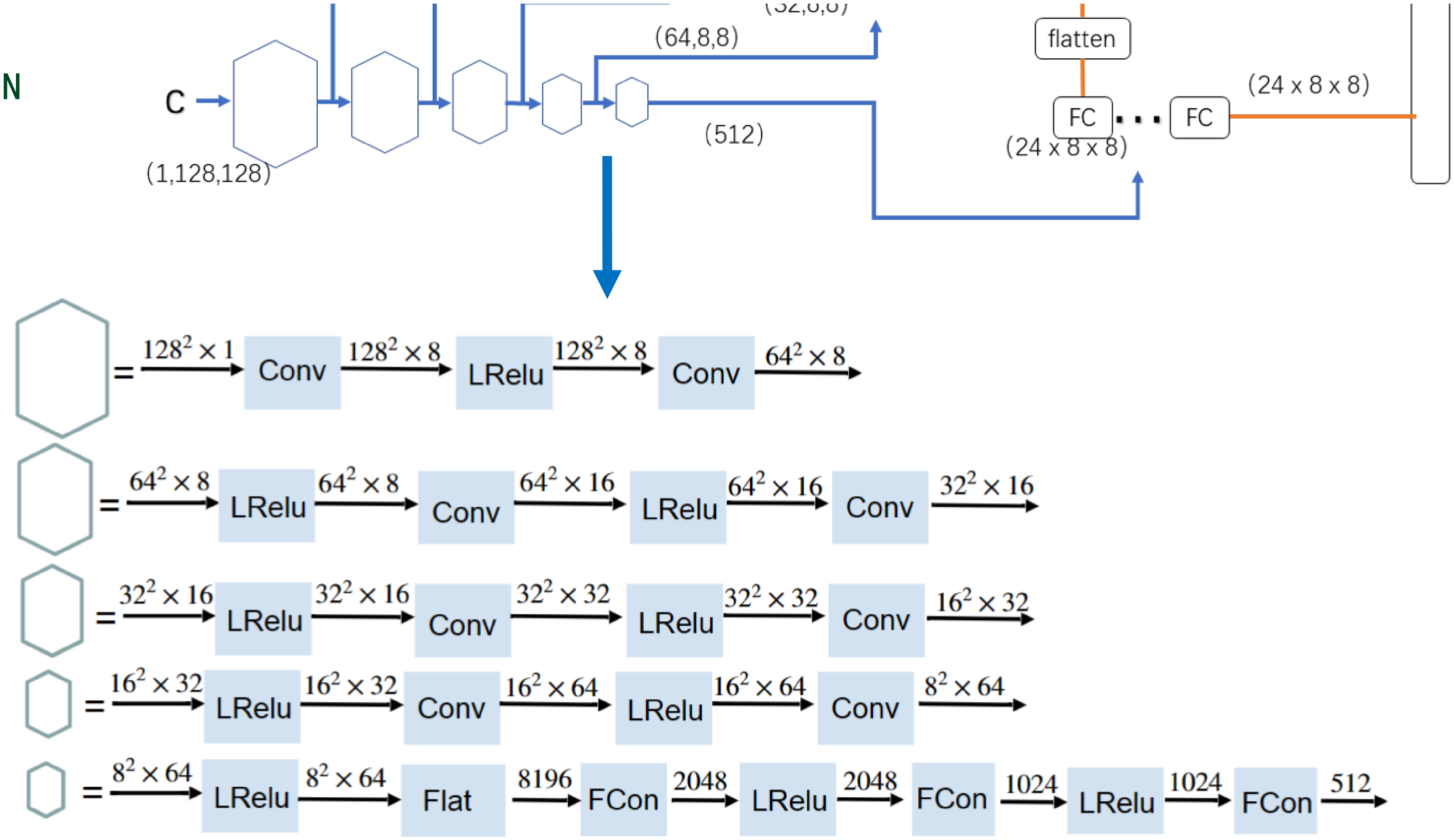
3.5 Structure of invertible neural network (I)

The dedicated network structure was found to reconstruct the sky map effectively



3.6 Structure of invertible neural network (II)

CINN



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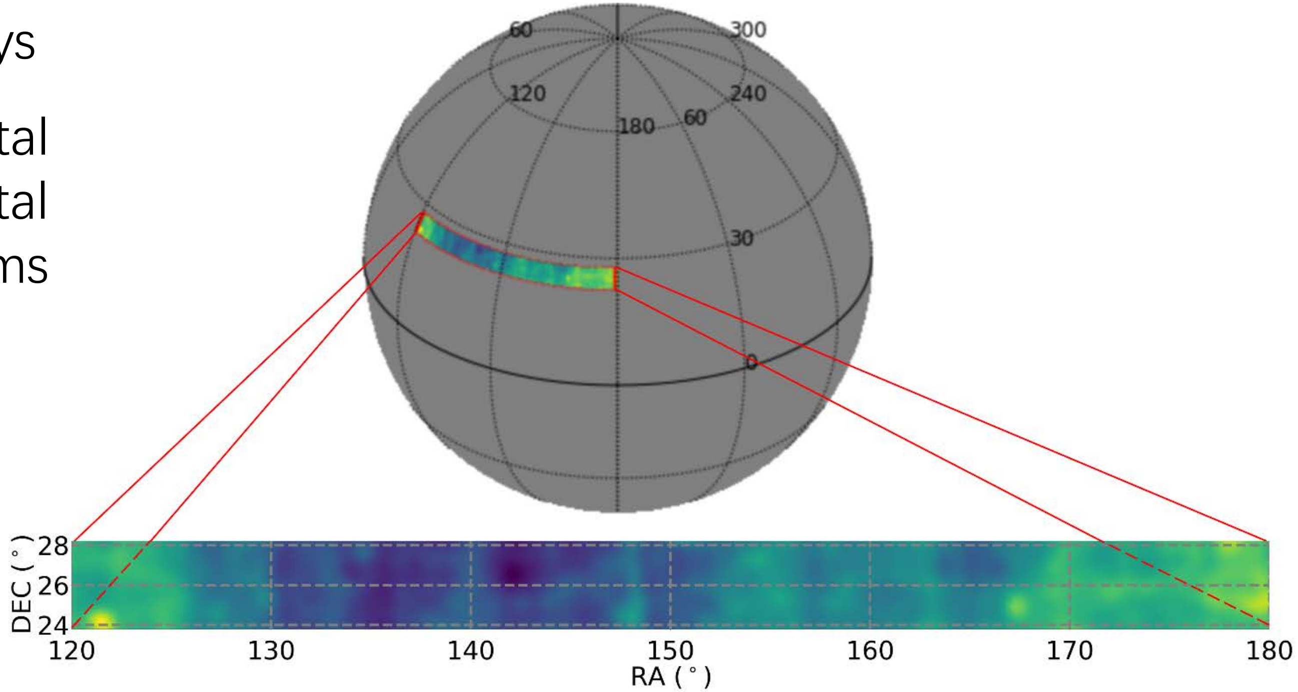
4.1 TOD generation and training samples

- **survey simulation**: based on the FAST configuration
- **drift scan:** consisting of a 19-beam receiver in the frequency range of 1100–1120 MHz, 20 channels
- **coverage:** a sky area of over 300 square degrees
- **observation time:** 2020.5.4–5.28, totaling 25 days
- an integration time of 1 s per beam and a total observation time of 14400 s/day, the total number of time samples for all 19 beams amounts to 25×14400×20×19 ~ 10^8
- **thermal noise** (T_{sys} in 0-25 K) added

- Data pre-possessing: due to the TODs' varying length and large data size, preprocessing is needed before feeding into the network
- TODs are gridded onto a 2D flat-sky maps, each having an area of $4.3° \times 4.3°$ and a resolution of 128 × 128



4.2 Evaluation Metrics

- the mean square error (MSE) : mean distance between two maps

$$\mathrm{MSE}\left(x_{\mathrm{true}}, x_{\mathrm{rec}}\right) = \frac{1}{N} \sum_{k=1}^{N} \left(x_{\mathrm{true}}^{k} - x_{\mathrm{rec}}^{k}\right)^{2}$$

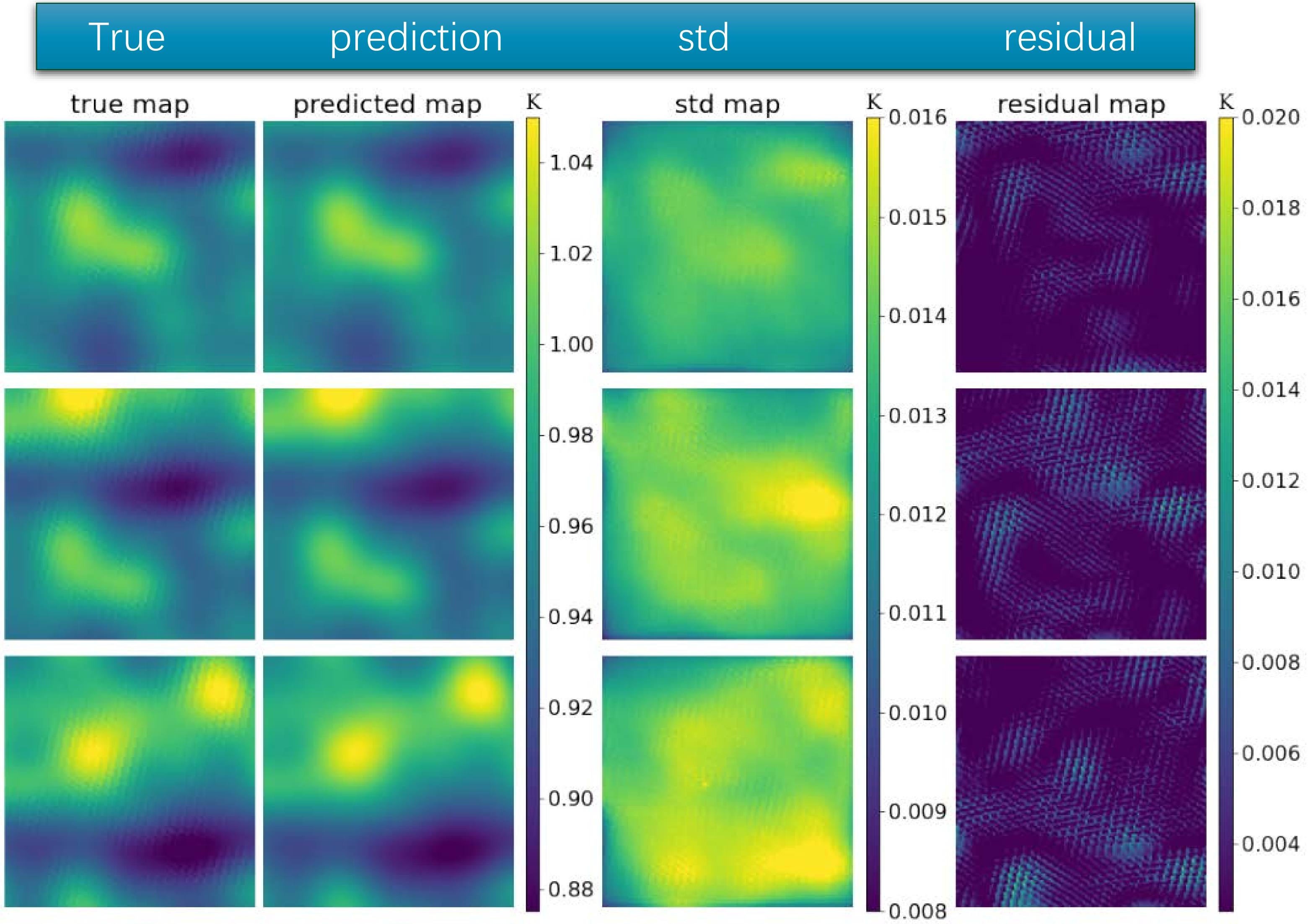
- the Peak Signal-to-Noise Ratio (PSNR): evaluating the reconstruction quality in dB

$$\mathrm{PSNR}(x_{\mathrm{true}}, x_{\mathrm{rec}}) = 10 \log_{10} \left(\frac{L^2}{\mathrm{MSE}\left(x_{\mathrm{true}}, x_{\mathrm{rec}}\right)} \right)$$

- the structural similarity index measure (SSIM)

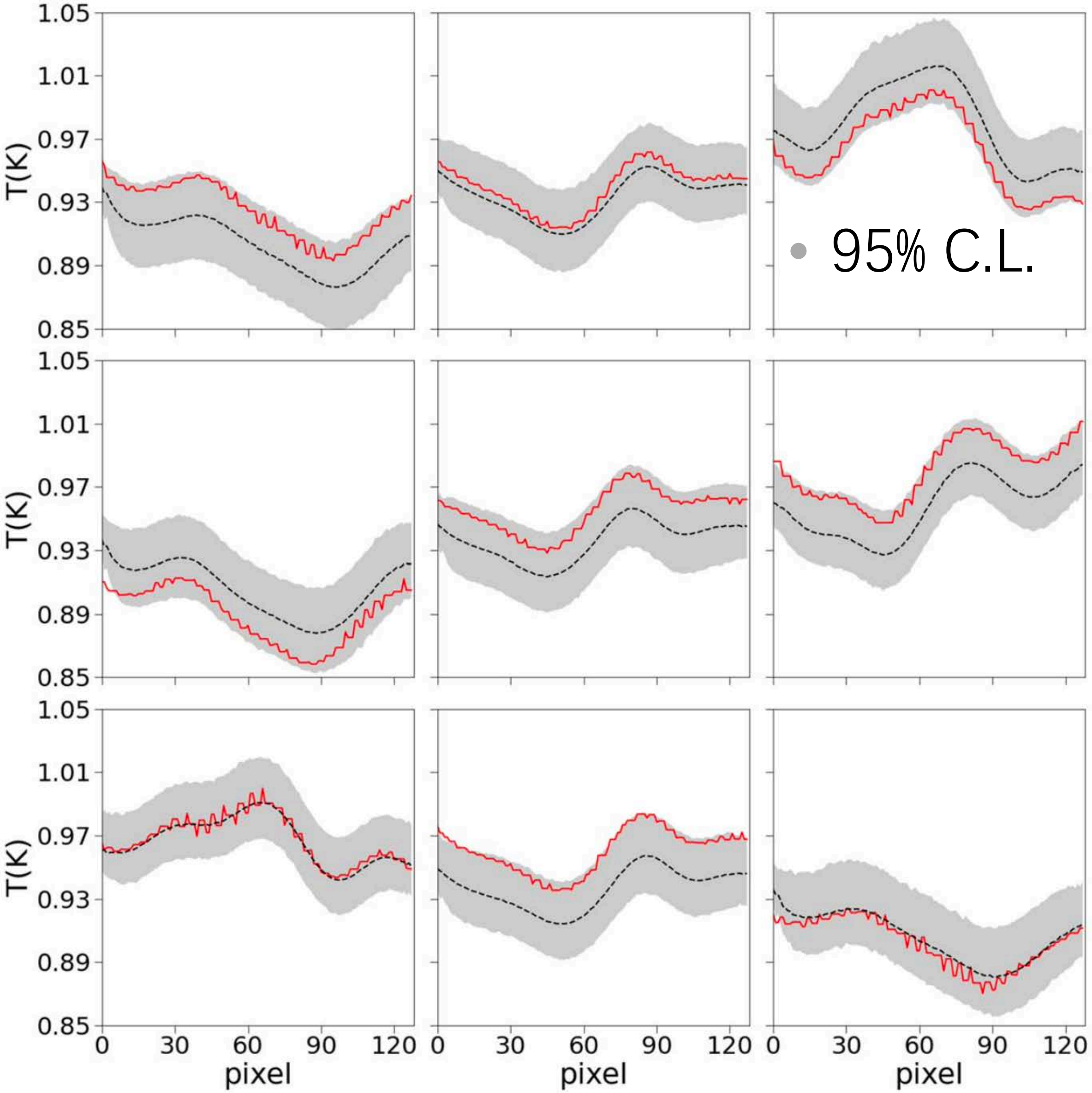
$$\mathrm{SSIM}(x_{\mathrm{true}}, x_{\mathrm{rec}}) = \frac{(2\mu_i\mu_j + C_1)(2\Sigma_{ij} + C_2)}{(\mu_i^2 + \mu_j^2 + C_1)(\sigma_i^2 + \sigma_j^2 + C_2)}$$

4.3 Results of map reconstruction



- $P(x|y)$ estimated by 200 reconstructed maps through drawing latent variables z from Gaussian
- std and residuals are about 0.01 K，1% level of true map
- good reconstruction quality

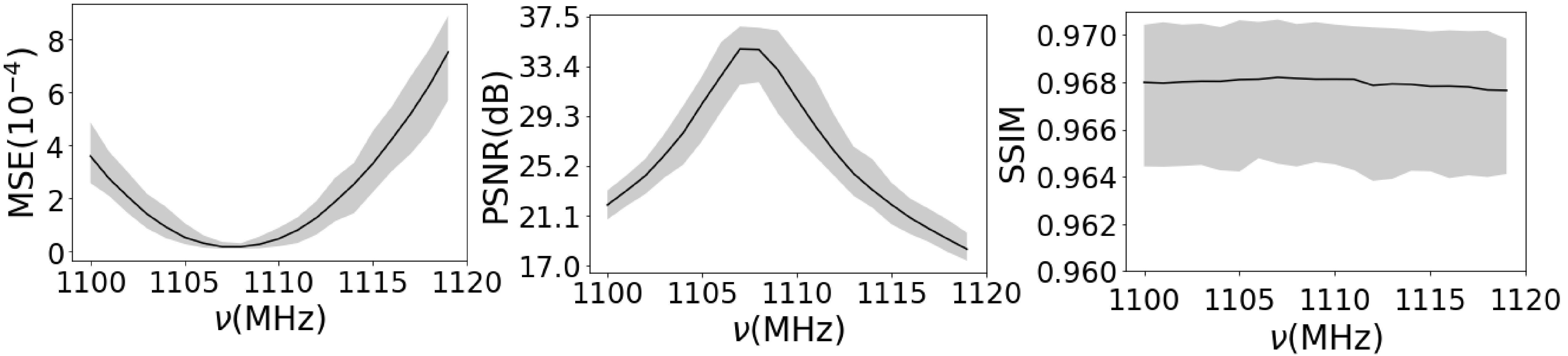
4.3 Results of map reconstruction



- randomly selected a row of map
- the mean vs. the true
- pixel error: 200 realizations of the latent variables z from normal distribution

Reconstruction mean and errors for each pixel can be precisely quantified.

4.4 MSE, PSNR, SSIM vs. frequency



over all test samples:

Performance	MSE ($\times 10^{-4}$)	SSIM	PSNR
	2.6 ± 5.0	0.95 ± 0.003	25.37 ± 4.21

- good performance in all three metrics across frequency

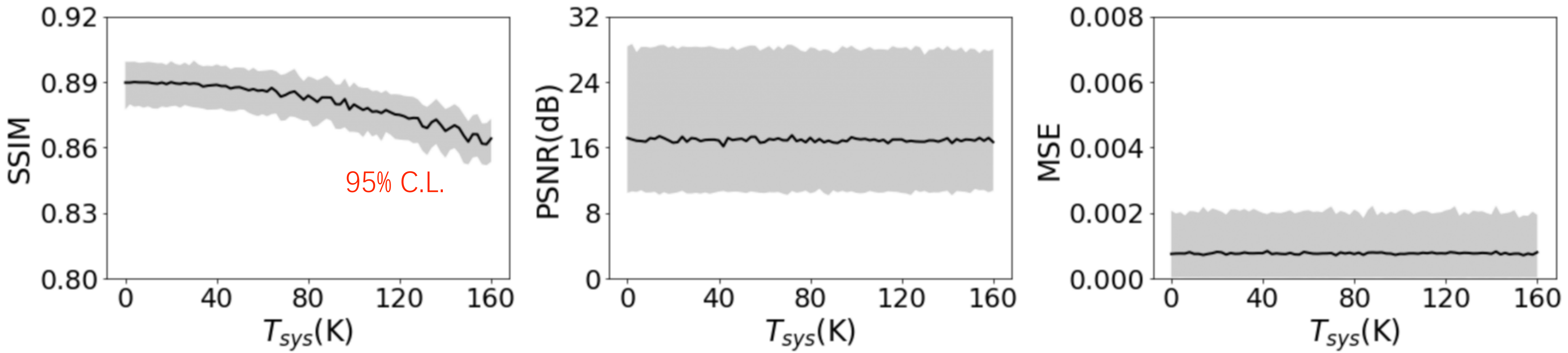
5. Summary and Outlook

- First introduce cINN to solve the map-making problem
- Good performance in reconstruction has been achieved
- cINN framework has the potential to tackle ill-posed problems in astronomy, like radio interferometric observations, where imaging can be particularly challenging due to sparse uv coverage

Backup

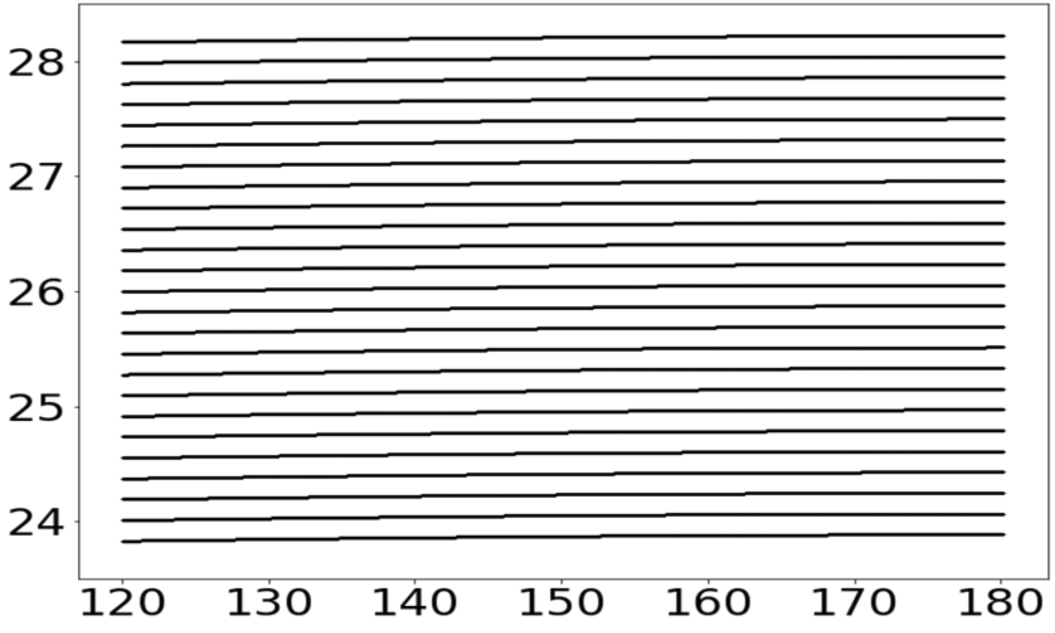
4.5 Performance against noise level

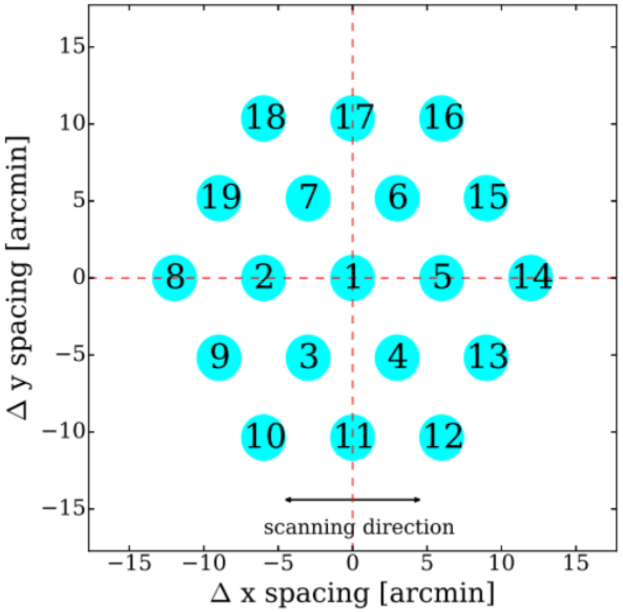
- T_{sys} in training： 0-25K; T_{sys} in generalization tests： 0-160K
- As the noise level increases, the SSIM value decreases from 0.89 to 0.85
- MSE and PSNR remain essentially constant with increasing noise level



2.1 Modeling of sky map reconstruction

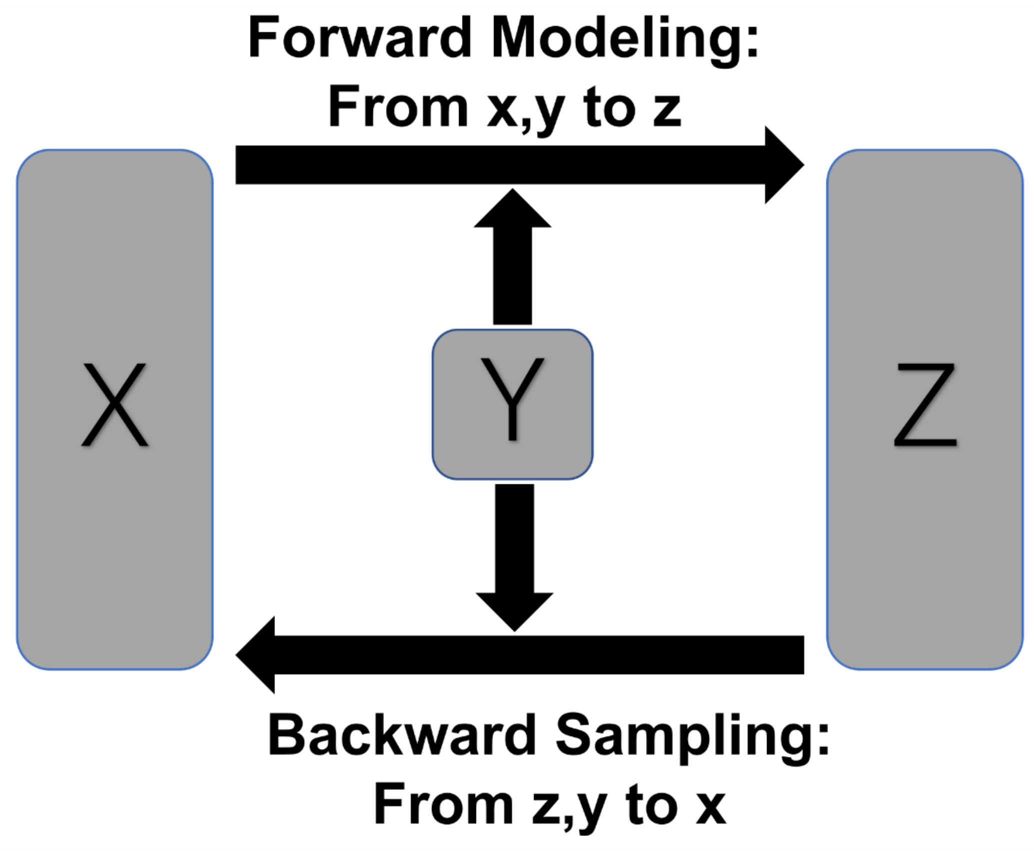
- **Observation equation:** $\vec{d} = A\vec{m} + \vec{n}$
  - m denotes sky map, d is for TOD, n is for noise
  - Dimension: d has N_t, m has N_m, A has $N_t \times N_m$, n has N_t
  - The Eq. can be extended to the multi-frequency and multi-beam case



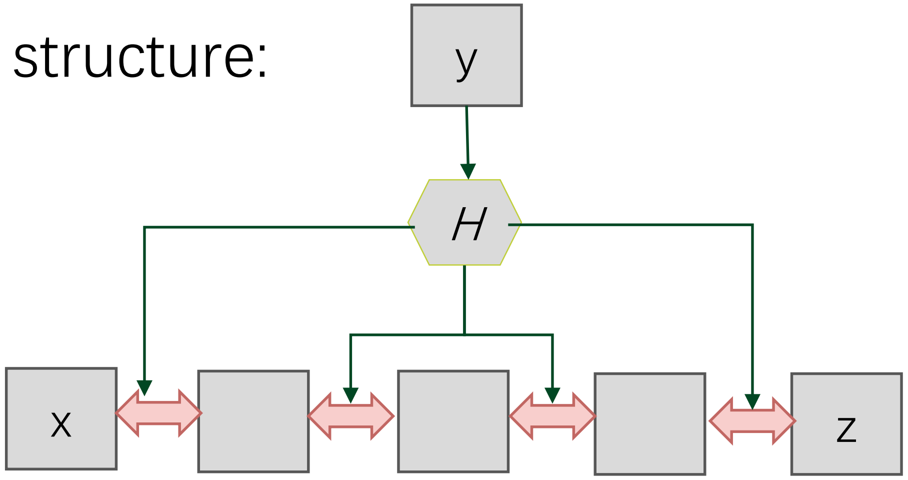


Hu (2021)

Main structure



Fully connected structure:



Multi-level structure: